THE ELECTRICAL MEASUREMENT OF MECHANICAL VIBRATIONS

by

Douglas S. Gordon, B.Sc., A.M.I.E.E., M.I.E.S.

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PREFACE

The work reported in this thesis was commenced in 1946 with a general survey of the literature and existing methods. Equipment was built up in order to test some original ideas for seismic detectors. In August 1947 we were invited by the British Shipbuilding Research Association to assist in a research programme to investigate the effects of entrained water on vibrating ships and ship models. This assistance was to take the form of advice about instrumentation and electronic measurement techniques. A range of special instruments was designed and constructed for the programme and developments have continued over a period of nine years. In the result we now have a set of instruments for producing vibration at a known and accurately controlled frequency and for measuring the vibration both in magnitude and phase. As commercial equipment was generally unsuitable for the purpose, this instrumentation has given substantial aid to the progress of the programme.

Thanks are due to the following:-

Professor B. Hague, for the use of the facilities of the Electrical Engineering Department, and for encouraging the long-delayed production of this thesis.

Messrs. A. Silverleaf and P.H. Tanner, who used the equipment at Leven Shipyard, Dumbarton, and Ship Division, National Physical Laboratory. Their comments and criticisms led to the development of apparatus which was adequate for dealing with the special problems of the research.

Mr. R. Smith, who constructed much of the final apparatus.

The British Shipbuilding Research Association, which sponsored the work and enabled the author to visit various establishments to discuss methods and apparatus with specialists in the field.

CONTENTS

Page No.

I. INTRODUCTION

1.1 Importance of Vibration with Particular Reference to Ships

It is improbable that any normal vibration which is at all tolerable could cause damage to the main structure of a ship, although damage to component parts, particularly under resonance conditions, can occur. However, as the comfort of crew and passengers is of considerable importance, it is desirable that vibrations which could affect their comfort do not occur. As it is very difficult and expensive in general to eliminate such vibrations after construction, it is necessary to predict the critical frequencies with accuracy at the design stage and to take suitable precautions to avoid exciting forces at these frequencies.

The calculation of the natural frequencies of a ship's hull is not easy for a number of reasons but, as far back as 1884, Schlick[1] concerned himself with the free-free vertical two-node vibration where the frequency f is given by the formula

$$f = \varphi\sqrt{\frac{I}{\Delta L^3}}$$

where Δ = displacement

L = length

I = moment of inertia of the midship section

φ = empirical coefficient.

There are a number of difficulties in the practical application of this formula, including lack of knowledge about certain of the quantities and, as a result, unless values are known for a very similar vessel, the formula is of little use.

Much work has been carried out since then to produce methods of predicting critical frequencies at the design stage. It is found, for example, that the frequencies are sensitive to the distribution of mass along the length of the ship but not very sensitive to the distribution of moment of inertia. On the other hand, the loading and draught have a pronounced effect, as has also entrained water and clearance between the bottom of the hull and the sea bed in shallow water[2].

It is apparent that the problem is complex and under these conditions, model tests are possibly capable of yielding useful results more readily than calculation.

1.2 Instrumentation

The measurement of mechanical vibrations by mechanical methods has been carried out for many years and much ingenuity has been devoted to designing detectors. However, most of these are of the seismic type and of considerable weight, particularly if suitable for measuring low frequency vibrations. Also, their upper frequency response is generally limited by the inertia of the mechanical lever systems and recording or indicating devices. Where a fixed reference is available, the problem is simplified but in this work, dealing with freely floating ship models which may heave, roll or pitch in addition to vibrating, no such reference is available. For these reasons, as well as convenience and general versatility, electrical methods of detection are necessary.

The vibrations had to be excited in the models in such a way that the condition of the model was changed as little as possible by external restraints or the mass of the exciter. Early tests indicated that the vibration modes of interest covered the frequency range of 5 to 200 c/s. For these reasons, it was decided that a mechanical exciter would be best as its force output per unit weight could be relatively large.

As the work progressed, further instrumentation in the form of speed stabilisation, phase measurement and automatic plotting units was developed. Where commercial equipment was available this was purchased.

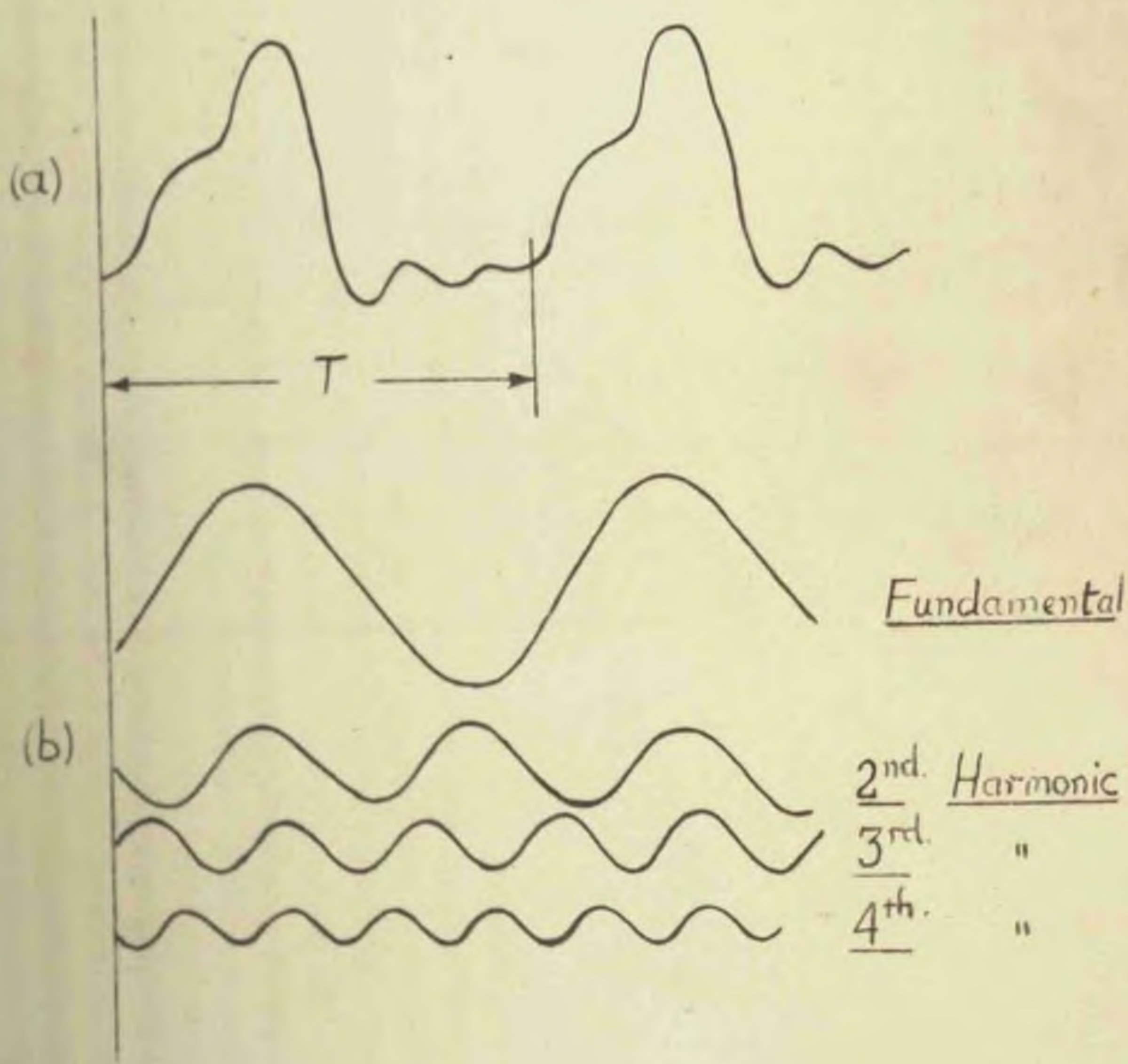

Fig. 1. (a) Complex waveform;

(b) Waveform of (a) analysed into its component parts.

2. THEORY

2.1 Vibratory Motion

A vibration is a periodic motion. Fig. 1(a) shows a typical plot of displacement to a base of time. After a period, T, the waveform repeats itself. This is a complex waveform, which may be analysed into harmonic components, Fig. 1(b). As all waveforms may be reduced to a number of sinusoidal components, a study of the conditions for each component will enable the conditions for the complex waveform to be synthesized.

Let the displacement x of a point on a vibrating body be represented by

$$x = x_o \sin \omega t \tag{1}$$

where x_o = amplitude of the vibration

and ω = angular frequency = $2\pi f$.

The period T is, therefore, given by $\frac{1}{f}$. For most mechanical purposes, it is convenient to employ a time unit of seconds so that f will be in cycles per second.

The velocity and acceleration of the point are obtained by differentiation, thus

$$\frac{dx}{dt} = \dot{x} = \omega x_o \cos \omega t \tag{2}$$

and
$$\frac{d^2x}{dt^2} = \ddot{x} = -\omega^2 x_o \sin \omega t = -\omega^2 x \tag{3}$$

The maximum or peak values of velocity and acceleration are thus ωx_o and $\omega^2 x_o$ respectively.

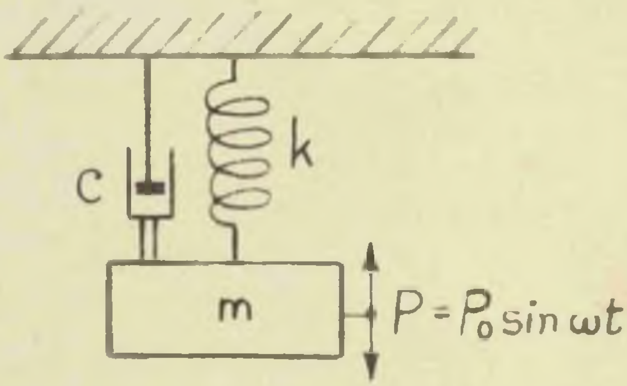

Fig. 2. Single-degree-of-freedom system with external exciting force $P = P_o \sin \omega t$.

2.2 Degrees of Freedom

A rigid body in free space has six degrees of freedom - three translational and three rotational - so that six co-ordinates are necessary to define its position uniquely. If the body can have relative motion between its parts, then the number of degrees of freedom is increased - up to infinity in a completely flexible system. The general analysis of such a system becomes impracticable and, for the purposes of this work, the dynamics of a system with one degree of freedom will be adequate.

2.3 Equation of Motion of Spring-Mass System with Damping

The components of a simple system, Fig. 2, may be considered as a mass m, a spring of stiffness k and a damping force due to friction or other losses which occur only when the mass is moving. It is convenient to assume that this damping force c is proportional to the velocity and acts in a direction opposite to the motion.

While the assumption of pure viscous damping is commonly made as it simplifies the mathematics, in practice many forms may occur, such as dry frictional which is independent of velocity or hydraulic which is proportional to the square of velocity. However, the viscous condition covers quite adequately the conditions to be considered here.

Any consistent system of units may, of course, be used but in this work the lb-in-sec system is convenient.

If a force $P_o \sin \omega t$ acts on the mass, and Newton's second law of motion, force = mass x acceleration, is applied, the following differential equation results -

$$m\ddot{x} + c\dot{x} + kx = P_o \sin \omega t \tag{4}$$

Making the right-hand side zero and solving, gives the general integral -

$$x = Ae^{\lambda_1 t} + Be^{\lambda_2 t} \tag{5}$$

where $$\lambda_1 = -\frac{c}{2m} + \sqrt{\left[\frac{c}{2m}\right]^2 - \frac{k}{m}} \tag{6}$$

and $$\lambda_2 = -\frac{c}{2m} - \sqrt{\left[\frac{c}{2m}\right]^2 - \frac{k}{m}} \tag{7}$$

The particular solution may be obtained in various ways, a neat method being that of Den Hartog[2]; giving

$$x = \frac{P_o \sin(\omega t - \varphi)}{\sqrt{\left[\omega^2 c^2 + (\omega^2 m - k)^2\right]}} \tag{8}$$

where $$\varphi = \tan^{-1} \frac{\omega c}{k - \omega^2 m} \tag{9}$$

The complete solution is thus given by the sum of equations (5) and (8). Equation (5) gives the motion of the mass under damped free vibration conditions and equation (8) the motion under forced vibration conditions.

Free Vibration

From the solution for the free-vibration condition it is seen that when

$$\frac{k}{m} > \left(\frac{c}{2m}\right)^2$$

$$\text{or} \quad c < 2m\sqrt{k/m} \tag{10}$$

the term under the square root is negative and, therefore, the motion is oscillatory but damped, that is, the amplitude of the vibration is decreasing with time.

$$\lambda = -\frac{c}{2m} \pm j\sqrt{k/m - \left(\frac{c}{2m}\right)^2} = a \pm jb \qquad (11)$$

$$\begin{aligned} \therefore \quad x &= Ae^{\lambda_1 t} + Be^{\lambda_2 t} \\ &= Ae^{(a + jb)t} + Be^{(a - jb)t} \\ &= Ae^{at}\, e^{jbt} + Be^{at}\, e^{-jbt} \\ &= e^{at}\left[A\,(\cos bt + j \sin bt) + B(\cos bt - j \sin bt)\right] \\ &= e^{at}\left[(A + B) \cos bt + j\,(A - B) \sin bt\right] \qquad (12) \end{aligned}$$

Thus the damped natural frequency of vibration f_n is given by

$$\omega_n = 2\pi f_n = b = \sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2} \qquad (13)$$

For large values of damping no oscillation takes place and the transition occurs when

$$\sqrt{\left(\frac{c}{2m}\right)^2 - k/m} = 0$$

$$\text{or} \quad c = 2\sqrt{mk} \qquad (14)$$

This value of c is called the critical damping c_c.

If there is no damping present, from equation (13), the angular frequency

$$\omega_o = \sqrt{k/m} \qquad (15)$$

$$\text{and} \quad \frac{\omega_n}{\omega_o} = \sqrt{1 - (c/c_c)^2} \qquad (16)$$

Forced Vibrations

In the absence of damping from equation (8)

$$x = \frac{P_o \sin \omega t}{\omega^2 m - k} \qquad (17)$$

so that x theoretically becomes infinite when

$\omega^2 = k/m$ or $\omega = \sqrt{k/m} = \omega_o$.

In practice, damping is always present and it is therefore convenient to plot response curves for a system with different amounts of damping. It is also convenient to plot these curves non-dimensionally so that they may be versatile in application. The abscissae are, therefore, the ratio ω/ω_o and the ordinates x_o/x_{stat}, where x_{stat} is the steady deflection of the spring due to the force P_o, that is, $x_{stat} = P_o/k$.

By substituting the values for c_c and ω_o derived above in equation (8) and rewriting

$$x = \frac{\left(P_o/k\right)\sin\left(\omega t - \varphi\right)}{\sqrt{\left[\left(2\frac{\omega}{\omega_o}\frac{c}{c_c}\right)^2 + \left(1 - \frac{\omega^2}{\omega_o{}^2}\right)^2\right]}} \tag{18}$$

or the amplitude of the displacement

$$x_o = \frac{P_o/k}{\sqrt{\left[\left(2\frac{\omega}{\omega_o}\frac{c}{c_c}\right)^2 + \left(1 - \frac{\omega^2}{\omega_o{}^2}\right)^2\right]}} \tag{19}$$

or $$\frac{x_o}{x_{stat}} = \frac{1}{\sqrt{\left[\left(2\frac{\omega}{\omega_o}\frac{c}{c_c}\right)^2 + \left(1 - \frac{\omega^2}{\omega_o{}^2}\right)^2\right]}} \tag{20}$$

which is the expression for the displacement amplitude in non-dimensional terms.

At resonance when $\omega = \omega_o$

$$\frac{x_o}{x_{stat}} = \frac{c_c}{2c} = Q \tag{21}$$

where Q is the "Magnification Factor" or "Quality Factor" of the system.

A number of curves are plotted for various values of the damping ratio c/c_c, Fig. 3.

The information given by the above expression is completed by plotting the phase relationship between force and displacement for the same range of conditions, Fig. 4.

From equation (9) -

$$\varphi = \tan^{-1} \frac{2(c/c_c)(\omega/\omega_0)}{1 - (\omega/\omega_0)^2} \qquad (22)$$

2.4 Analogies

It is often useful to be able to compare electrical and mechanical systems and, where both systems are present in the one unit, as for example, in a gramophone pick-up and reproducer system, the overall response can be computed if the electrical and mechanical constants are known.

In a series circuit consisting of resistance, inductance and capacitance, R, L and C respectively, supplied from a source of e.m.f. $E_0 \sin \omega t$, the current i is given by the expression,-

$$L \frac{di}{dt} + Ri + \frac{1}{C} \int i.dt = E_0 \sin \omega t \qquad (23)$$

If the charge $q = \int i.dt$ is substituted in this equation

$$L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + \frac{1}{C} q = E_0 \sin \omega t$$

$$\text{or} \quad L \ddot{q} + R \dot{q} + \frac{1}{C} q = E_0 \sin \omega t \qquad (24)$$

which is directly equivalent to the mechanical equation of motion already derived.

This analogy is given in detail in Table 1.

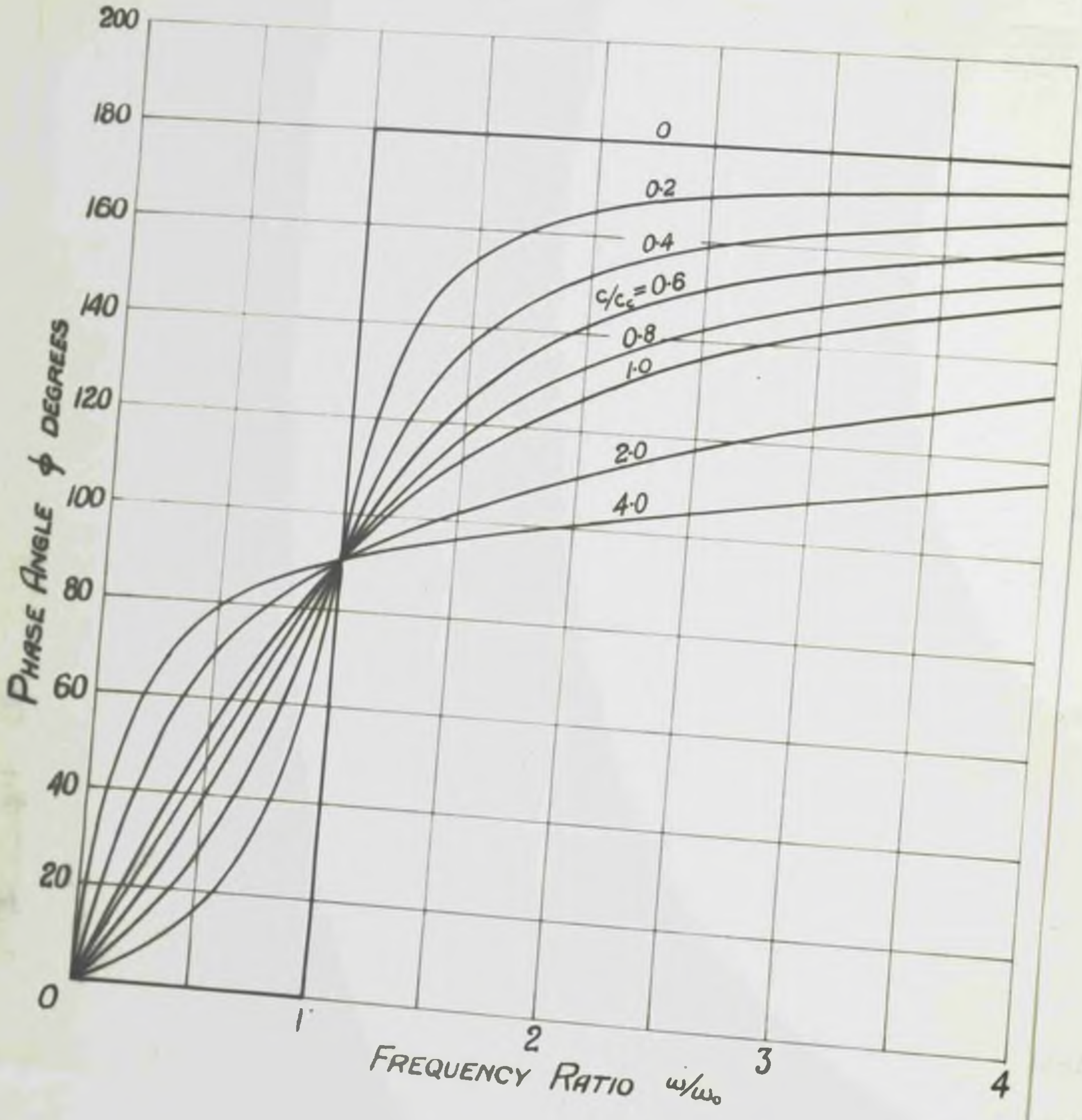

Fig. 4. Phase angle between the exciting force and the resulting displacement for various values of damping.

TABLE 1

Mechanical		Electrical	
Force	P	E.M.F.	e
Deflection	x	Charge	q
Velocity	$\dot{x}$	Current	i
Mass	m	Inductance	L
Stiffness	k	$\frac{1}{\text{Capacitance}}$	$1/C$
Damping	c	Resistance	R
Potential Energy	$\frac{1}{2} kx^2$	Electrostatic Energy	$\frac{1}{2} \cdot \frac{1}{C} q^2$
Kinetic Energy	$\frac{1}{2} m\dot{x}^2$	Electromagnetic Energy	$\frac{1}{2} Li^2$
Mechanical Impedance	$Z = P/\dot{x}$	Electrical Impedance	$Z = e/i$

While the above analogy is commonly used, it has been pointed out that, as electrical series and mechanical parallel combinations are being compared, serious difficulties may be introduced when it is applied to complex systems.

A very useful summing-up of various analogies and a reasonably comprehensive bibliography is given in a paper by Prache[4]. It appears that the series analogy due to Firestone[5] has much to recommend it although initially strange and apparently cumbersome.

The equivalent systems are shown in Fig. 5 and it is seen that if the force P acts through the mechanical system (corresponding to current i) and if p is an impulse given by $\int Pdt$, then

$$\frac{1}{k} \ddot{p} + \frac{1}{c} \dot{p} + \frac{p}{m} = \dot{x} \tag{25}$$

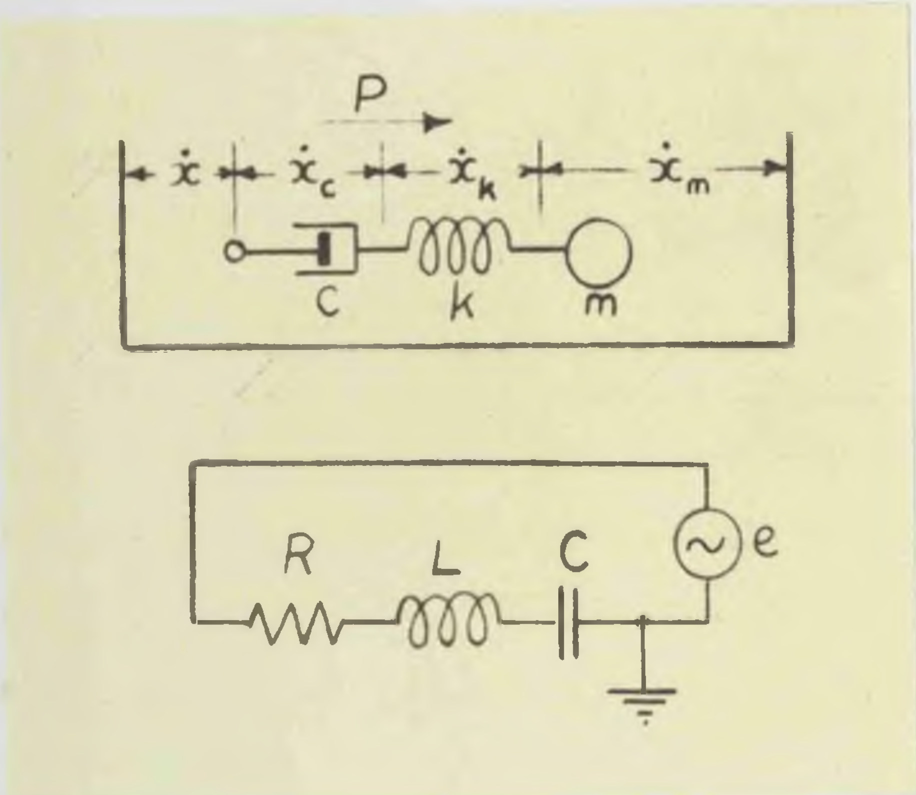

Fig. 5. Electro-mechanical analogy according to Firestone.

A table of electrical and mechanical equivalents may now be drawn up[6] and it becomes apparent that this analogy has advantages over the conventional one, particularly in complex systems where combinations of mechanical impedances in series and parallel have to be evaluated.

Some consideration was given at one time to the possible construction of an electrical analogy of the ship model with provision for adding equivalent "mass" to allow for the effect of entrained water. Due to the relative complexity of the system, no practical action was taken.

3. MEASUREMENT OF VIBRATION

As mentioned in Section 2.2, a rigid body has six possible degrees of freedom, so that a complete determination of the motion of a point on the body would require a measurement of the displacement of the point with time for each of the translational and rotational freedoms. The resultant motion of the point could then be computed from the six components. Fortunately, a vibrating member is usually constrained so that the number of components which make substantial contributions to its motion is relatively small.

3.1 Methods of Measurement

The measurement of a vibration may be attempted in four distinct ways, namely:

3.1.1 Direct measurement of displacement using a fixed reference station.

This technique has the advantage of being absolute and, where practicable, is very often the simplest. Its limitations are obvious in the case of ships afloat, where additional motion is superimposed on the vibration to be measured. These limitations may be overcome to some extent by the use of filters in the measuring equipment, but the method was not considered satisfactory in this particular application.

3.1.2 Measurement of fluctuating strain at points in the structure and hence computation of the vibration conditions.

This system has certain advantages and has been used extensively in many problems involving the determination of stresses in

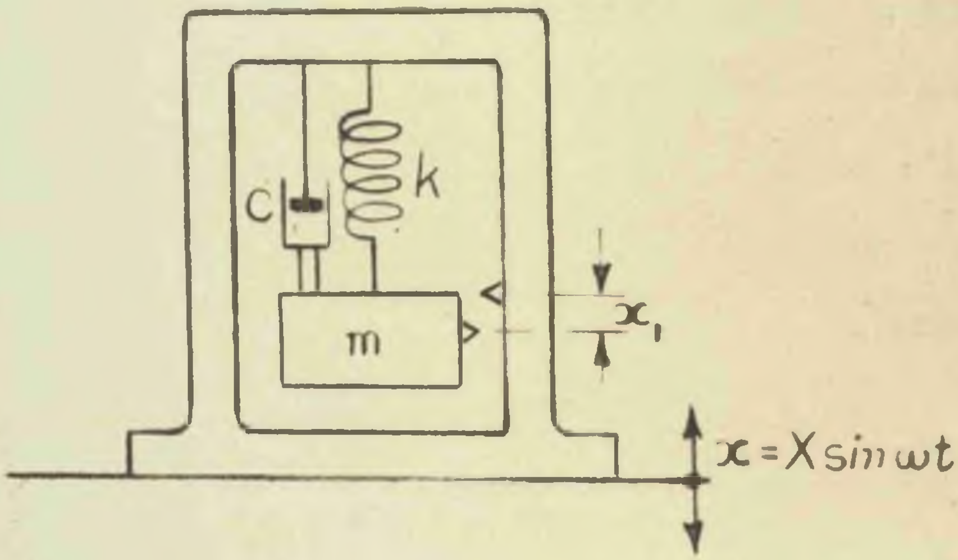

Fig. 6. Elements of a vibrometer.

hulls and airframes. However, it is often difficult to determine absolutely the magnitudes of the vibrations and to arrange that different modes of vibration can be isolated and measured.

3.1.3 <u>A spring-and-mass system whose natural frequency is lower than the frequency of the vibrations to be measured</u>.

If the device shown in Fig. 6 is placed on a body with a vertical component of vibration, the mass m tends to remain stationary in space and hence provide a fixed reference. In order to extend the useful frequency range and to reduce the transient effects, damping may be introduced. Such a unit is termed a seismic detector.

3.1.4 <u>A spring-and-mass system whose natural frequency is higher than the frequency of the vibration to be measured</u>.

The diagrammatic representation of this unit will be the same as that for 3.1.3., but in this case, the mass will have a dynamic displacement which is proportional to the vertical acceleration of the unit. Damping must be introduced to reduce transient effects, to correct the output for any harmonic content of the vibration and to extend the useful frequency range of the instrument. This form of detector is termed an accelerometer.

From the above brief comparison of methods of measurement, it will be apparent that either 3.1.3 or 3.1.4. could form the basis for a suitable vibrometer. A simple analysis of their dynamic characteristics is therefore given.

3.2 Spring-Mass Vibrometer Characteristics

The elements of the vibrometer are given in Fig. 6 and consist of a spring-mass system with damping contained in a rigid box, which is mounted on the vibrating body. The analysis is kept simple by assuming a perfectly elastic spring and viscous damping.

Let the displacement of the vibrating body be sinusoidal and given by $x = X \sin \omega t$, where $\omega = 2\pi f$ and f is the frequency of vibration.

If the relative displacement between the seismic mass m, and the vibrometer box is instantaneously x_1, then the equation of motion of the system may be written:

$$m(\ddot{x} + \ddot{x}_1) + c\dot{x}_1 + kx_1 = 0 \quad , \tag{26}$$

where c and k represent the damping and spring constants respectively.

As before, the undamped natural frequency of the spring-mass system $f_o = \omega_o/2\pi$ is given by:

$$\omega_o = \sqrt{k/m} \qquad \text{or } \omega_o^2 = k/m$$

and since

$$x = X \sin \omega t$$

$$\dot{x} = \omega X \cos \omega t$$

and $$\ddot{x} = -\omega^2 X \sin \omega t$$

Further, if sufficient damping is introduced so that the free vibration just becomes aperiodic, then, this amount of damping, given before as the critical damping

$$c_c = 2\sqrt{km} = 2\omega_o m \qquad \text{or } \frac{1}{m} = \frac{2\omega_o}{c_c}$$

Rewriting the equation of motion and substituting these values:

$$\ddot{x}_1 + 2\omega_0(c/c_c)\dot{x}_1 + \omega_0^2 x_1 = \omega^2 X \sin \omega t \qquad (27)$$

which has the complete solution

$$x_1 = e^{-(c/c_c)\omega_0 t}\left[A \sin \omega_n t + B \cos \omega_n t \right]$$

$$+ \frac{X \left(\frac{\omega}{\omega_0}\right)^2 \sin(\omega t - \varphi)}{\sqrt{\left[\left(1 - \frac{\omega^2}{\omega_0^2}\right)^2 + \left(2\frac{c}{c_c}\cdot\frac{\omega}{\omega_0}\right)^2\right]}} \qquad (28)$$

where $\omega_n = \omega_0 \sqrt{1 - (c/c_c)^2}$ (29)

and $\varphi = \tan^{-1} \dfrac{2(c/c_c)(\omega/\omega_0)}{1 - (\omega/\omega_0)^2}$ (30)

The first term of the solution, containing the arbitrary constants A and B which depend on the initial conditions, represents the transient response and may be neglected in the analysis of the behaviour of an instrument subjected to steady-state conditions. With reasonable damping, the effective output of the vibrometer due to any such transient is short-lived.

From the second term of the solution, the displacement of the seismic mass relative to the box is seen to be sinusoidal and to have a peak value

$$X_1 = \frac{X \left(\frac{\omega}{\omega_0}\right)^2}{\sqrt{\left[\left(1 - \frac{\omega^2}{\omega_0^2}\right)^2 + \left(2\frac{c}{c_c}\ \frac{\omega}{\omega_0}\right)^2\right]}} \qquad (31)$$

This motion in general lags the vibration by the angle φ whose value is given in equation (30) above, which is the same expression as was given in equation (22).

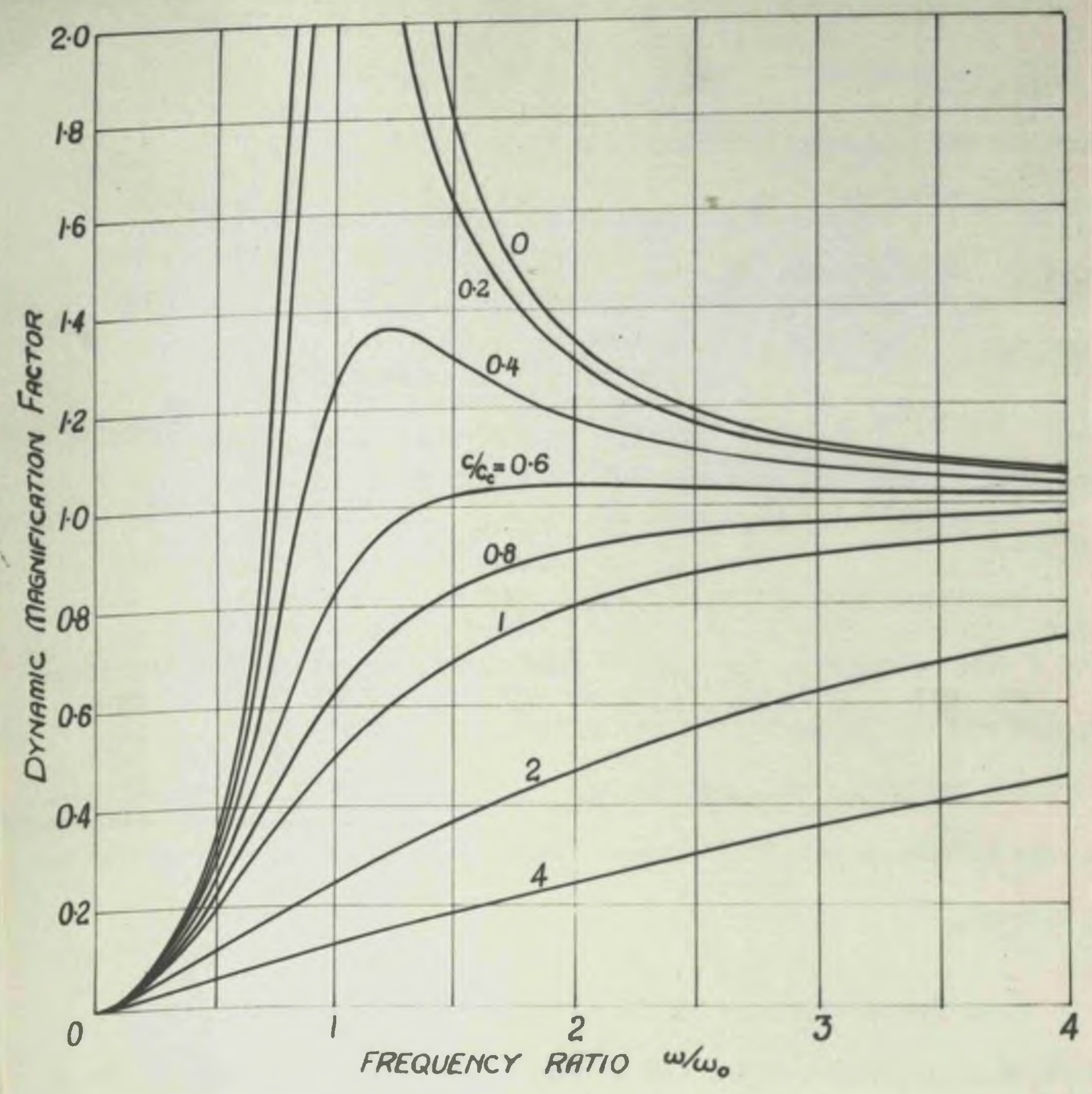

Fig. 7. Dynamic magnification factor for vibrometer.

The instrument dynamic magnification factor

$$Q = \frac{X_1}{X} = \frac{\omega^2/\omega_o^2}{\sqrt{\left[(1 - \omega^2/\omega_o^2)^2 + (2\{c/c_c\}\{\omega/\omega_o\})^2\right]}} \tag{32}$$

Curves of Q plotted to a base of frequency ratio ω/ω_o for various values of damping ratio are shown in Fig. 7.

It can be seen from these curves that a seismic instrument, whose natural frequency is less than one-third of the lowest vibration frequency to be measured, will indicate true amplitude with small error for a wide range of damping coefficient. This accuracy, however, is attained at the expense of a relatively large phase error, which may vary considerably if the frequency range is large. Curves of phase angle plotted to a base of frequency ratio ω/ω_o for various values of damping ratio have been given in Fig. 4.

On the other hand, for vibration frequencies appreciably less than the natural frequency of the instrument, it can be seen from Fig. 7 that the dynamic magnification factor is approximately parabolic in form

$$\text{or } Q = \frac{X_1}{X} \propto \omega^2 \tag{33}$$

But if $x = X \sin \omega t$

$$\ddot{x} = -\omega^2 X \sin \omega t = -\omega^2 x$$

$$\therefore \quad Q \propto x \tag{34}$$

That is, the response of the instrument is proportional to the vibration acceleration. The actual response departs from the ideal by an amount which depends on the damping and frequency ratios. From equation (31) this is seen to be

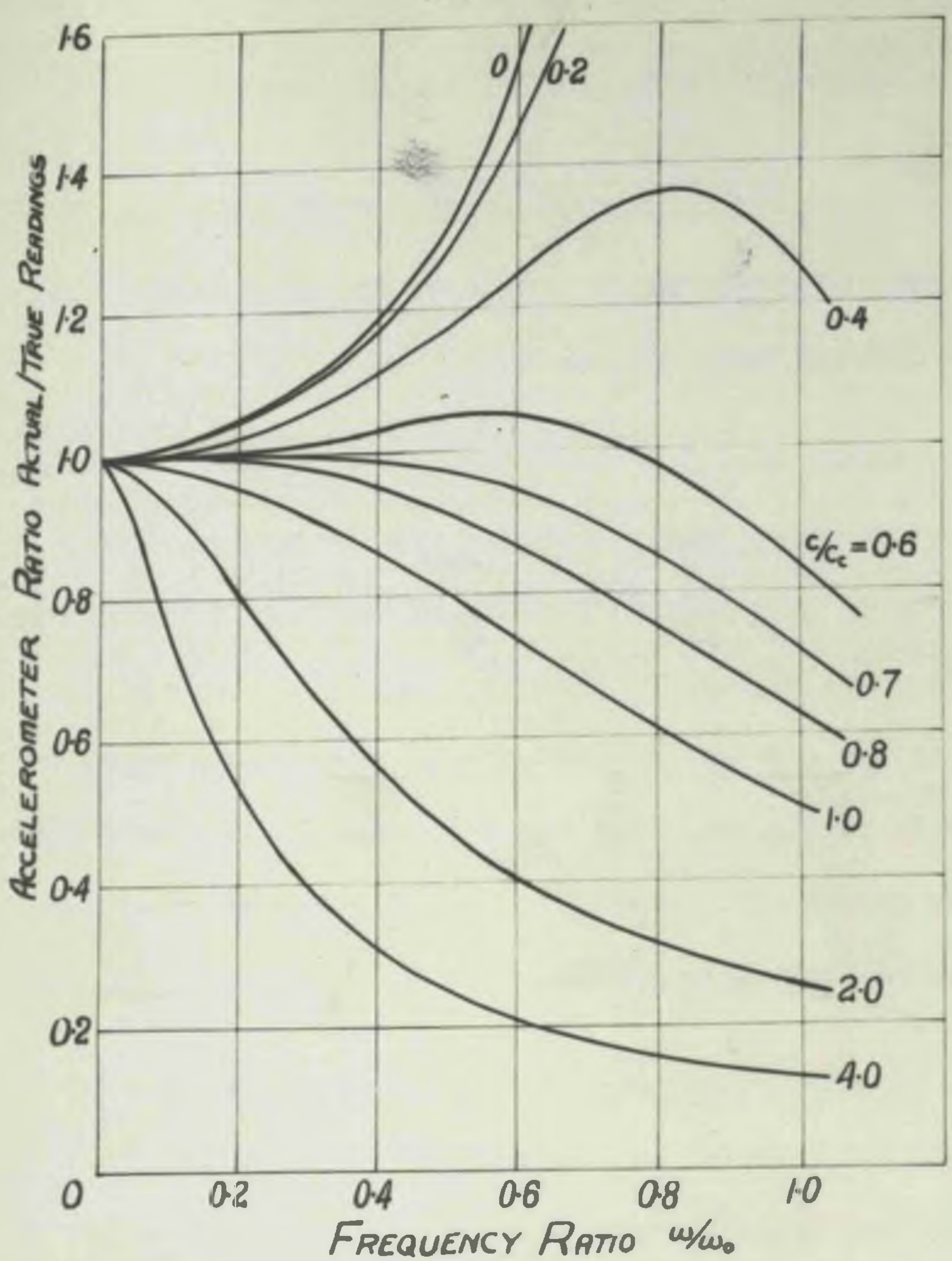

Fig. 8. Accelerometer response showing effect of viscous damping.

$$\text{Ratio} \quad \frac{\text{actual reading}}{\text{true reading}} = \frac{1/\omega_o^{\,2}}{\sqrt{\left[(1 - \omega^2/\omega_o^{\,2})^2 + (2\ c/c_c\ \frac{\omega}{\omega_o})^2\right]}} \qquad (35)$$

A family of curves, showing this relationship, is plotted in Fig. 8. It can be seen that errors less than 5% over a frequency range of 0 to 0.8 ω_o are obtainable if the damping is about 0.6 of the critical value. This value of damping is about the optimum and substantial errors can arise, particularly at the higher vibration frequencies, if there is any appreciable departure from this value.

3.3 Damping

Most designs for accelerometers require damping additional to that occasioned by hysteresis in the spring material or friction in pivots or other forms of energy dissipation. In commercial units this damping is commonly obtained by filling the box containing the spring-mass system with a silicone oil. This has the advantages of relative cheapness and a viscosity not too dependent on temperature. However, if the unit is to be used in a wide range of ambient temperatures, some care must be taken if large errors are to be avoided. Typical response curves for a commercial accelerometer of resonant frequency 65 c/s and nominal damping 0.65 c_c at 20°C are shown in Fig. 9. A more constant damping factor could be obtained by employing a dashpot system with a valve adjustable either manually or automatically to compensate for the change in viscosity of the fluid. Generally speaking, such a refinement would be difficult to design and hardly worth the additional complication.

In some forms of pick-up employing powerful permanent magnets eddy-current damping is readily arranged. This appears to be very satisfactory and much less dependent on temperature. However, if the magnet is not already part of the unit, it is again doubtful if the advantages of this type of damping generally justify its installation.

Sponge rubber, soft plastic and similar materials are used to some extent, but no figures are available for their practical performance and it is to be expected that they are temperature sensitive and, in some cases, severely affected by ageing. In addition, their damping force probably departs considerably from the ideal.

3.4 Vibrometer Output

The output from seismic and accelerometer vibration detectors is mechanical in the first instance. This output is generally in the form of a displacement of an element of the detector relative to the casing or spring support and may be observed directly as such. For small amplitudes, a mechanical lever or an optical pointer system may be employed to obtain a greater movement. The waveform of the vibration may be recorded by pen on paper, scratch on celluloid or the effect of the light beam on photographic film. While a purely mechanical instrument of this type has many uses and can yield valuable results, its scope is generally limited and a more versatile instrument results if the mechanical displacement is converted into an electrical signal, which may be amplified, operated on, read at a distance, or recorded in one or more of several ways for future reference. The conversion is performed in a mechano-electric transducer, or transducer for short.

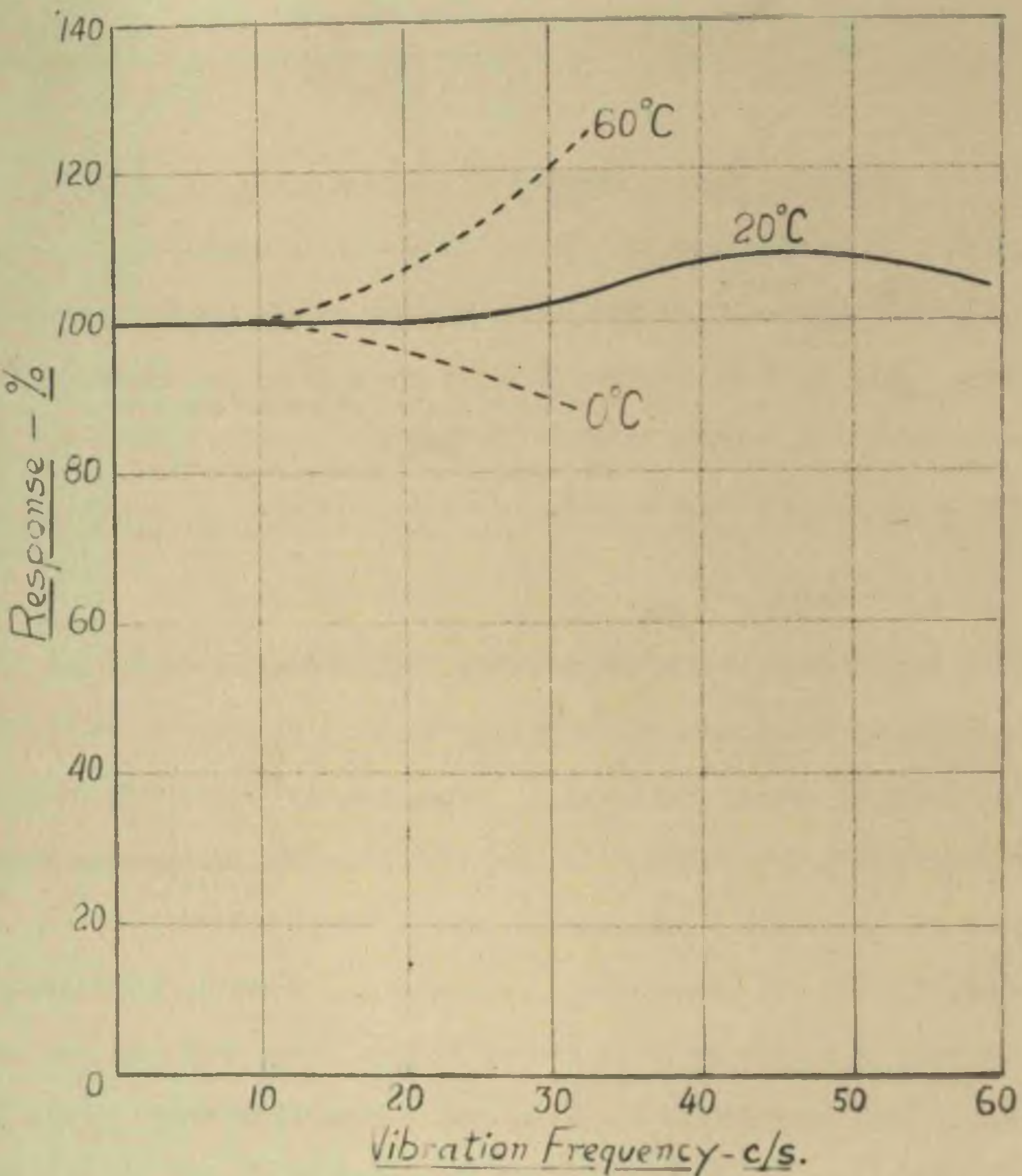

Fig. 9. Response curve for accelerometer at different ambient temperatures.

Sample of foil strain gauge.

4. TRANSDUCERS

The possible forms of this device are many and a few of the more common varieties which are relevant to the work and of which the author has had some experience are described.

While a three-phase alternator is a transducer by the above definition, it is improbable that it would make a suitable energy convertor for inclusion in a vibration detector. It is, therefore, convenient to list the general requirements which may have to be satisfied.

Reliable.

Output a linear function of input.

Zero (or constant) phase difference between output and input signals with varying frequency.

Output independent (or a simple function) of frequency for a given signal level.

Large "transduction factor", that is, large output signal for a given input, so that very high gain amplifiers following the transducer are unnecessary and a high signal/noise ratio may be more readily obtained.

Free from difficult electric supplies (that is, supplies highly stable in frequency or voltage).

Economical in consumption so that small batteries may be used for portable equipment.

Light.

Robust.

Cheap.

Easy to use and maintain.

Simple.

The relative importance of any of the above factors depends on the particular application, and no one system can satisfy all requirements completely.

4.1 Strain Gauges

These may be of the bonded or unbonded types. In the former, a thin metallic wire or foil is made in the form of a flat grid and cemented to a base of paper or plastic. Further protection is generally given to the unit by a top covering of paper, plastic sheet or varnish. The gauge is attached to the object being tested by some type of adhesive, which must produce an exceptionally good bond if the gauge is to follow exactly the deformation of the object.

In the unbonded type, the wire is stretched in air between two insulators which are connected rigidly to the object being tested.

The resistance of the gauge is

$$R = \frac{\rho l}{A} \tag{36}$$

where ρ = specific resistance of the wire or foil (commonly constantan ρ = 49 $\mu\Omega$cm)

l = total length of the wire or foil

A = cross-sectional area of the wire or foil.

Thus if the gauge is strained

$$\frac{dR}{R} = \frac{dl}{l} - \frac{dA}{A} \tag{37}$$

for a circular wire of diameter D, $A = \frac{\pi D^2}{4}$

$$\therefore \frac{dR}{R} = \frac{dl}{l} - 2\frac{dD}{D} \tag{38}$$

According to Poisson's Law

$$\frac{dD}{D} = -\mu \frac{dl}{l} \tag{39}$$

where μ = Poisson's Ratio

= 0.33 for constantan

$$\therefore \frac{dR}{R} = \frac{dl}{l}(1 + 2\mu) \tag{40}$$

Thus the gauge factor or sensitivity constant

$$k = \frac{dR}{R} \div \frac{dl}{l} = 1 + 2\mu \tag{41}$$

= 1.66 for constantan.

In fact, for reasons which are not very clear, but may be due to changes in the specific resistance with strain, the gauge factor commonly has a value of about 2.

4.1.1 Temperature Effects

For most practical purposes the change in resistance of the gauge due to strain is small and the change due to temperature change is comparable. Where the gauge is being used to measure vibrations, the temperature effect is of little consequence but for slow variations in strain, account must be taken of temperature variation during the period of test. This is conveniently done by having a second similar gauge close to the measuring gauge but not subject to load. This second dummy gauge can be connected in the resistance measuring circuit to compensate for variations in resistance due to temperature.

A further temperature effect, which may introduce error in the measurement of slowly varying strain is the differential linear coefficient of expansion between the gauge material and the material of the body or structure under test.

The coefficients of expansion of mild steel and constantan are $11.35 \times 10^{-6}/^{\circ}C$ and $17.0 \times 10^{-6}/^{\circ}C$ respectively, so that the error due to this effect is small where the range of temperature is moderate. Also, the resistance temperature coefficient of constantan is variously quoted in the range of $-0.4 \times 10^{-4}/^{\circ}C$ to $+ 0.1 \times 10^{-4}/^{\circ}C$ so, again, compensation by means of a dummy gauge is generally adequate for normal measurements.

4.1.2 Application to Vibrometer

In the pick-up, the gauges may be applied to leaf springs supporting the mass. With a gauge above and below each of two springs, all four arms of the bridge may be strain gauges giving automatic temperature compensation and greater sensitivity[7].

By supporting a mass in a form of "cat's cradle" made up of unbonded strain gauges, an accelerometer can be constructed to give three outputs corresponding to the translational accelerations in the x-y-z directions.

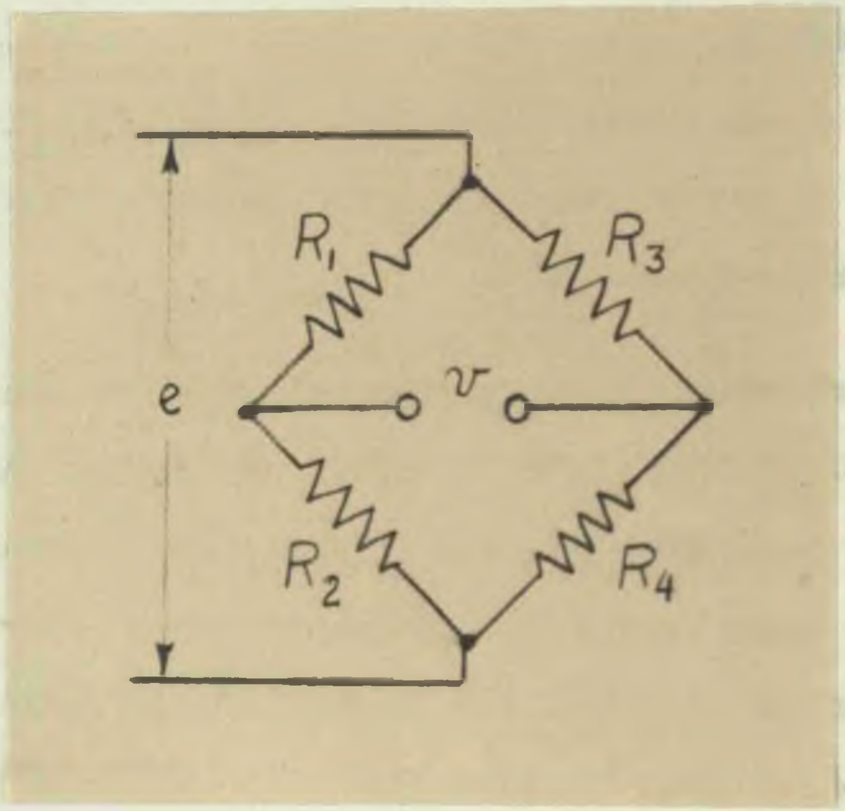

Fig. 10. Simple resistance bridge.

4.1.3 Sensitivity of a Wheatstone Bridge

The arms of a simple resistance bridge, Fig. 10, for strain gauge measurements have values in the range 100 to 2000Ω. The detector may be a sensitive galvanometer or a more robust instrument preceded by an electronic amplifier. In either case, for an approximate analysis, it is assumed that the detector input impedance is infinite. Hence

$$v = \left[\frac{R_1}{R_1 + R_2} - \frac{R_3}{R_3 + R_4}\right] e \tag{42}$$

If R_1 is the resistance of the active gauge and it suffers a small change of resistance ΔR_1, the resulting change in output voltage

$$\Delta v \doteq e \frac{R_2 \Delta R_1}{(R_1 + R_2)^2} \tag{43}$$

Thus, for small resistance changes, there is an approximately linear relationship between the output voltage and the strain.

Optimum sensitivity occurs when R_1 is substantially equal to R_2, so that

$$\Delta v \doteq e \frac{\Delta R_1}{4R_1} \tag{44}$$

Greater sensitivity results if two or all four arms of the bridge are active gauges, Section 4.1.2.

4.2 Differential Transformer

Various forms of this type of transducer have been described in the literature and some are available commercially. Fig. 11 shows diagrammatically the unit which was used as a detector in the assembly described in Section 15. By employing a relatively large magnetic flux, an output sufficient to operate a rectifier instrument without amplification was easily obtained with some sacrifice of linearity. To obtain an output proportional to displacement, the maximum displacement must be small compared to the air gap.

Better linearity over a wider range of displacements can be obtained by arranging the coils coaxially with a common cylindrical moving core. One such unit[8] is stated to be linear within $\pm$ 1% for movements up to 0.1" and to give in the most sensitive range, full-scale deflection of the indicator for a movement of 0.0001". It would probably be a little difficult to design a satisfactory vibrometer around this unit.

If the waveform of the vibration is of interest, it is necessary to insert a "polarising" signal in series with the transducer output as the output is independent of the sense of the movement. If the transducer output is zero in the rest position, the arrangement is equivalent to the "suppressed carrier" system in communications and a carrier must be re-introduced to avoid the "double-frequency" effect. Circuits for phase-sensitive detectors are given in the literature and need not be discussed here. Fig. 12 gives a simple circuit, which is adequate for rough measurement purposes.

4.3 Variable Inductance

There are a number of commercial manufacturers of this type, which can have various forms. In one of these, a spring-mounted core of

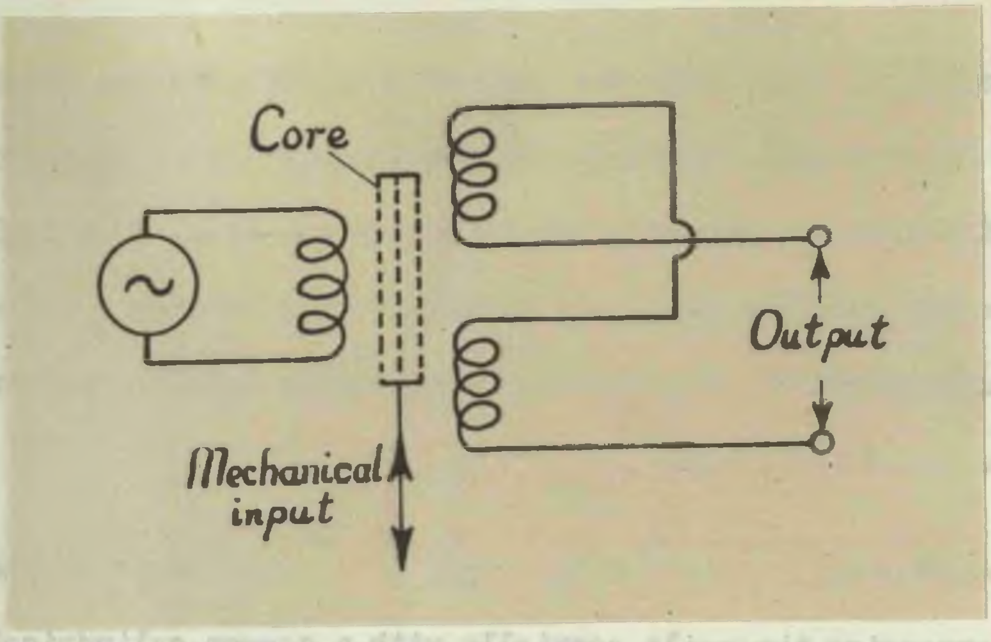

Fig. 11. Moving-armature differential-transformer transducer.

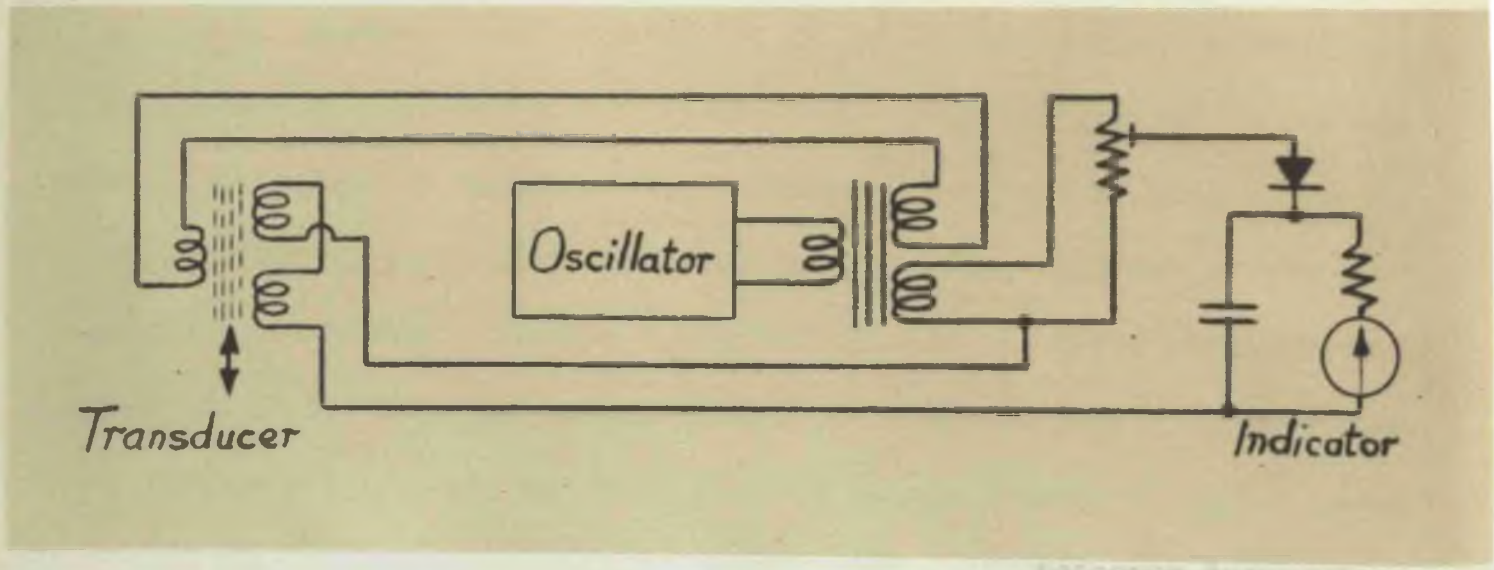

Fig. 12. Phase-sensitive detector.

Ferroxcube or other suitable material is arranged so that, when the unit is subjected to acceleration, the core moves and the inductance of a coil in proximity to the core changes. In an improved version, there are two coils arranged so that movement of the core produces equal and opposite changes of inductance. The inductances are approximately equal when the core is in the rest position.

Some experience has been gained with a Lan-Elec Transducer, Type I.T. 1 - 22, illustrated in Plate I(b). This unit is of the balanced construction described above and has the following characteristics:-

Weight	$2\frac{1}{2}$ oz
Supply	10V at 2000 c/s (maximum current in coils 6 mA)
Coil Inductance	70 mH $\pm$ 10 mH
Natural Frequency	100 c/s
Damping coefficient	0.8

The damping is produced by filling the unit with silicone oil so that "c" does not vary excessively with temperature. The unit is claimed to have a constant output with a given acceleration for frequencies up to 50 c/s. Tests on the vibrating table showed that this was true within $\pm$ 5%. Further tests showed that the output was proportional to acceleration up to its maximum rated value of $\pm$ 5g (1930 in/sec^2). The effect of temperature was not investigated.

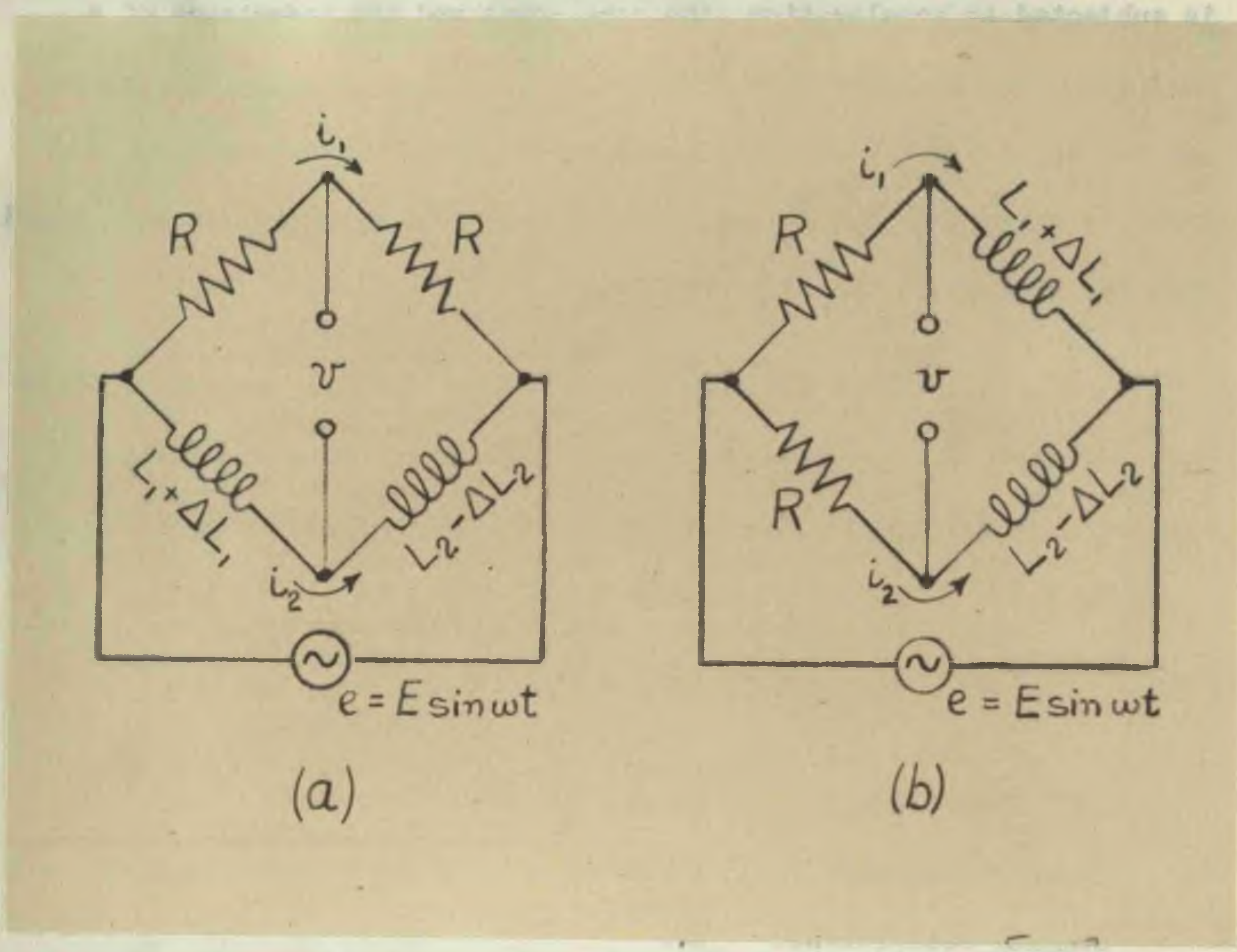

Fig. 13. Inductance bridges.

A phase-shift oscillator, amplifier and phase-sensitive detector were constructed following conventional circuits with minor modifications but, as the crystal pick-up in use was found to be adequate, these units were not put to practical use.

4.3.1 Bridge Sensitivity

The bridge connections may be as shown in Figs. 13(a) or (b). The sensitivity conditions can be readily derived, assuming that the external resistors R are equal and that $L_1 = L_2 = L$ in the rest condition. When the transducer is subjected to an acceleration, the core is displaced and L_1 is increased by an amount ΔL_1 and L_2 decreased by an amount ΔL_2. Ideally, these increments are equal in magnitude. An infinite impedance detector in the form of a valve amplifier is normally used.

Connection (a)

$$i_1 = \frac{e}{2R} \qquad (45)$$

$$i_2 = \frac{e}{j\omega\,(2L + \Delta L_1 - \Delta L_2)} \qquad (46)$$

$$\therefore\ v = \frac{e}{2R} \times R - \left[\frac{e}{j\omega\,(2L + \Delta L_1 - \Delta L_2)}\right] \times \left[j\omega\,(L - \Delta L_2)\right]$$

$$= e\left[\frac{1}{2} - \frac{L - \Delta L_2}{2L + \Delta L_1 - \Delta L_2}\right]$$

$$= e\left[\frac{2L + \Delta L_1 - \Delta L_2 - 2L + 2\Delta L_2}{2\,(2L + \Delta L_1 - \Delta L_2}\right]$$

$$= e\left[\frac{\Delta L_1 + \Delta L_2}{2\,(2L + \Delta L_1 - \Delta L_2)}\right]$$

$$\doteq e\left[\frac{\Delta L_1 + \Delta L_2}{4L}\right] \qquad (47)$$

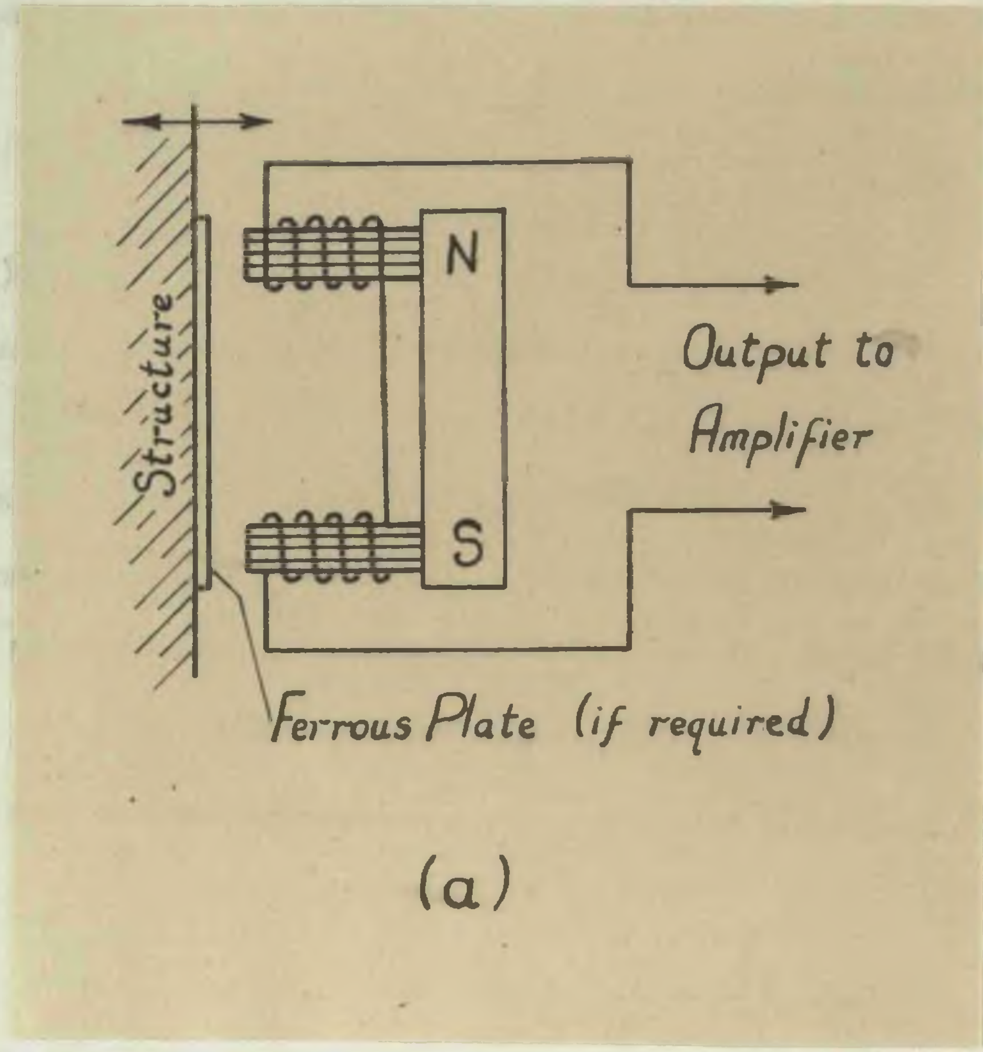

Fig. 14. Telephone headpiece, moving-iron electromagnetic transducer.

or, assuming $|\Delta L_1| = |\Delta L_2| = \Delta L$

$$v/e \doteq \frac{\Delta L}{2L} \tag{48}$$

Connection (b)

$$i_1 = \frac{e}{R + j\omega(L_1 + \Delta L_1)} \tag{49}$$

$$i_2 = \frac{e}{R + j\omega(L_2 - \Delta L_2)} \tag{50}$$

Making the same assumptions as before, and in addition, taking the condition for maximum sensitivity, namely, $R = \omega L$

$$v \doteq e\,\frac{\Delta L_1 + \Delta L_2}{2L} \tag{51}$$

$$\text{or } v/e \doteq \frac{\Delta L}{L} \tag{52}$$

It follows that this connection gives approximately twice the output for a given acceleration compared to the former connection.

In practice, the analysis is not quite so simple, as the inductances and their increments are not necessarily equal. Resistance is associated with the inductance and it can be shown that, if the coil has an impedance $(r + j\omega L)$, the optimum sensitivity is obtained when $R^2 = r^2 + \omega^2 L^2$.

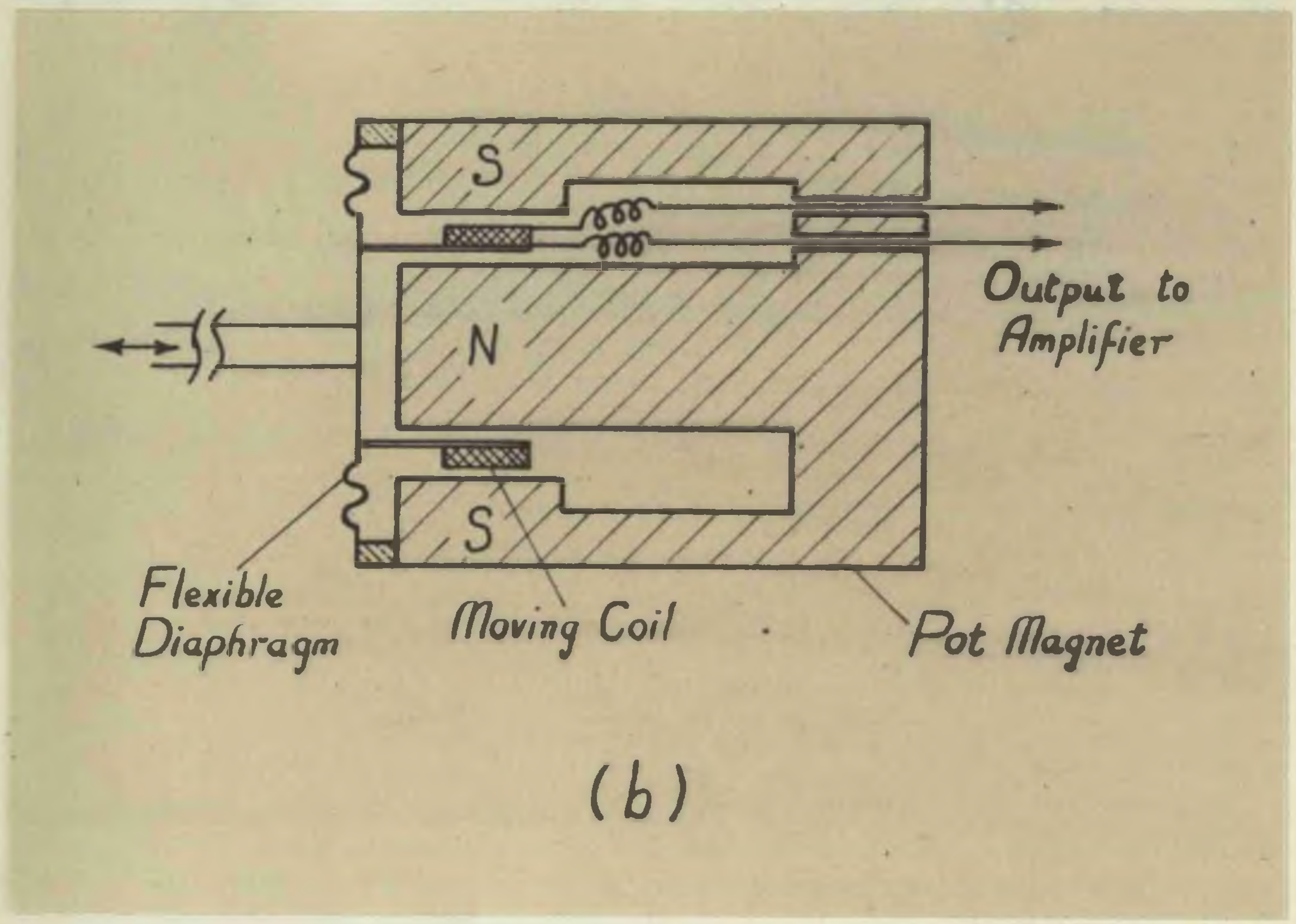

Fig. 15. Moving-coil electromagnetic transducer.

4.4 Variable Reluctance

Where a very simple qualitative or a rough quantitative test is required, an ordinary telephone headpiece, Fig. 14, is very useful and easily rigged in certain circumstances. The output, which is approximately proportional to velocity if the displacement is small compared to the air gap, may be heard without amplification in headphones or amplified to operate some form of indicator. The author has used this successfully in the tests described in Section 13 and also to look for vibrations in marine turbine reduction gearing[9].

If the air gap is $x = A_o (1 + \Delta \sin \omega t)$ and there is a magnetic flux Φ corresponding to the gap A_o, then the generated e.m.f.

$$\begin{aligned} e &= k\frac{d\Phi}{dt} = k\frac{d\Phi}{dx} \cdot \frac{dx}{dt} \\ &= k\frac{d\Phi}{dx} \cdot \frac{d}{dt}\left[A_o \; (1 + \Delta \sin \omega t)\right] \\ &= k\frac{d\Phi}{dx}\, \omega \Delta A_o \cos \omega t \end{aligned} \tag{53}$$

Hence the output is proportional to velocity provided $\frac{d\Phi}{dx}$ is constant, which is nearly true if $\Delta \ll 1$.

The mean air gap A_o chosen is a compromise between sensitivity and linearity.

4.5 Moving Coil

The moving-coil generator, Fig. 15, can be designed to give an output signal which is accurately proportional to the relative velocity between the coil and the field magnet and which is independent of coil position for a wide range of displacements. For these reasons, it is a

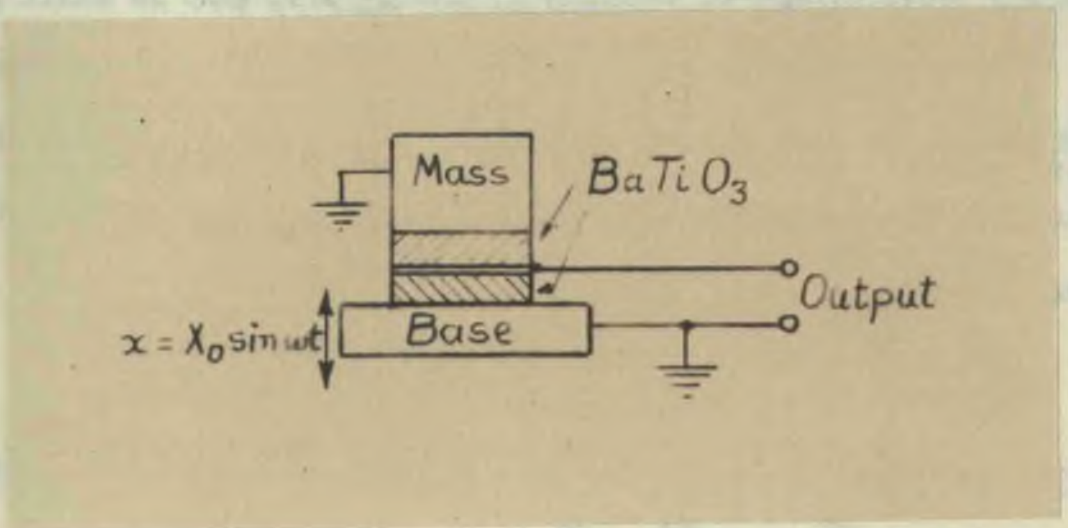

Fig. 16. Piezo-electric accelerometer using barium titanate discs.

popular form of transducer in many commercial pick-ups. Electro-magnetic damping which is accurately proportional to velocity and not greatly dependent on temperature, can also be readily incorporated. The precise workmanship necessary to produce such units, particularly if they are small and light, tends to make them relatively expensive.

The initial tests on the wax ship models were carried out with a seismic detector of this type. It, however, was large and cumbersome, weighing about 15 lb. Its natural frequency was about 3 c/s and a certain amount of eddy-current damping occurred in the steel core of the moving coil. Due to its large mass, its use was discontinued as soon as more suitable equipment became available.

4.6 Piezo-Electric

Certain materials such as quartz, Rochelle salt and barium titanate produce an electric charge when strained. This charge is proportional to the strain and hence may be used as a measure of the strain. Because of the nature of these materials, the piezo-electric effect is normally used only in accelerometer type pick-ups where the natural frequency of the system can be high. Fig. 16 shows diagrammatically a typical unit employing barium titanate discs.

4.6.1 Barium Titanate

Widely varying claims have been made for titanate elements but an authoritative source gives the following constants for a material containing 96% $Ba\,Ti\,O_3$ and 4% $Pb\,Ti\,O_3$ polarised at 750 V/mm at 140°C

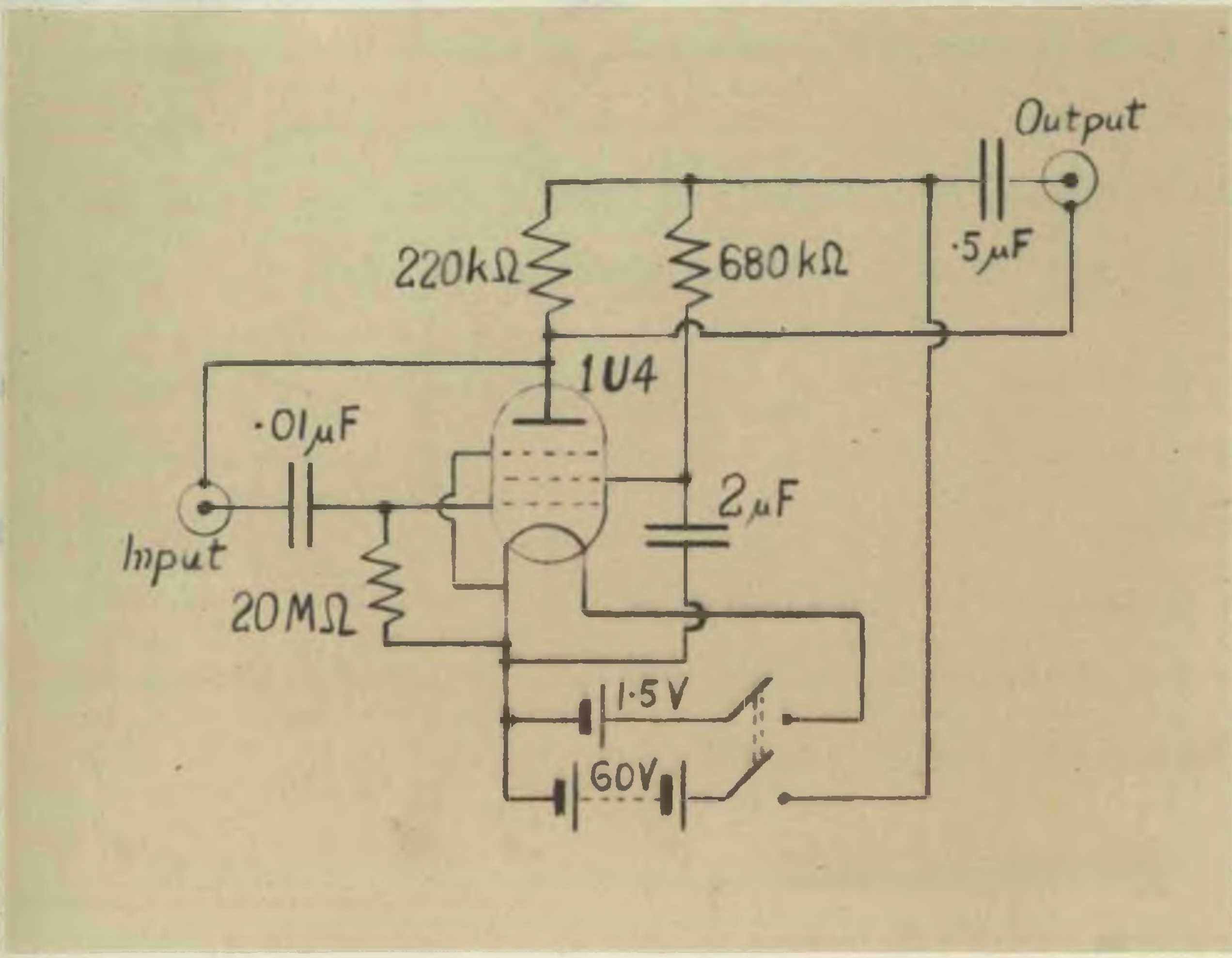

Fig. 17. Circuit for impedance converter (after circuit contained in N.B.S. Report No. 2654).

Charge sensitivity Q_s = 0.66×10^{-15} coulomb/dyne
Permittivity = 1200 - 1500

The crystal output is shunted by the input impedance of the amplifier, so that a compromise between sensitivity and low-frequency response has to be sought.

For a unit of capacitance 400 pF carrying a mass of 1.4 gm the open-circuit output voltage, using the above material, is

$$v = \frac{Q_s mg}{C} = \frac{0.66 \times 10^{-15} \times 1.4 \times 981}{400 \times 10^{-12}} = 0.0022 \text{ V/g} \qquad (54)$$

The resonant frequency is about 37 kc/s[10].

Calculation shows that the output is down by 30% at 800 c/s with an amplifier input resistance of 0.5 MΩ, but that the equivalent low frequency response is extended to 40 c/s if the input impedance is increased to 10 MΩ.

In order to obtain a good l.f. response, a head amplifier with a very high input impedance is necessary. A suitable battery-operated cathode follower circuit is given in Fig. 17. By situating this unit close to the pick-up, the shunting effect of the cable capacitance is minimised and the upper frequency response is also improved.

4.6.2 Rochelle Salt

The pick-up used in most of the tests employed a Rochelle salt crystal contained in a heavy die-cast housing sealed to prevent the ingress of moisture, Plate I(C). Little information was obtainable from the manufacturers about its construction or characteristics, except that the maximum acceleration should not exceed 7g. When used with a Dawe Vibration Meter, Type 1402C, the response is claimed to be within ± 10% from 4 to 1000 c/s. Tests on the vibrating table showed that it was within these limits over the frequency range 10 to 80 c/s.

4.6.3 Other Materials

The composition and properties of titanate elements are very variable but, in general, the advantage of the high charge sensitivity is nullified by the high permittivity. Rochelle salt is more sensitive but is limited in use by its temperature and moisture characteristics. A number of other materials have been investigated, and a good case has been made for using strontium formate dihydrate which has a charge sensitivity/permittivity ratio about three times that of Rochelle salt[11].

However, as in the case of the titanate elements, a great deal of development work will probably be necessary before these special materials can be used in reliable and consistently accurate accelerometers.

4.6.4 Screened Cable

Standard screened or coaxial cable often exhibits a pseudo piezo-electric effect, which may introduce relatively large signals into a high-impedance circuit. This effect is possibly due to the development of electric charges on the surface of the insulation next to the braided outer conductor. Various cures are possible, including a thin graphite deposit on the dielectric surface.

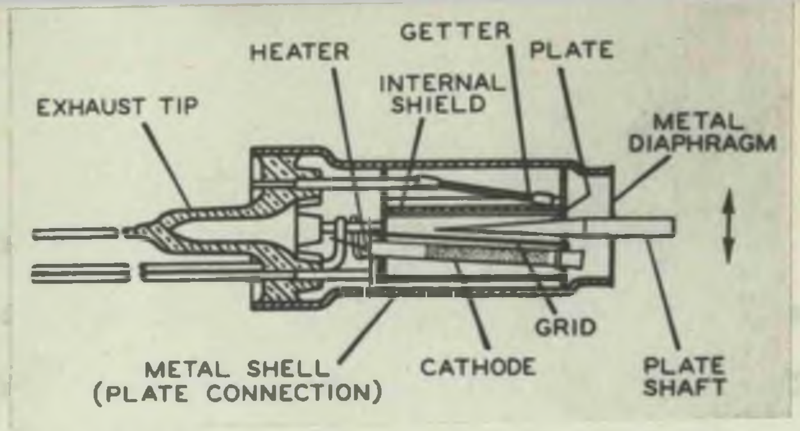

Fig. 18. Section of R.C.A. Transducer Triode.

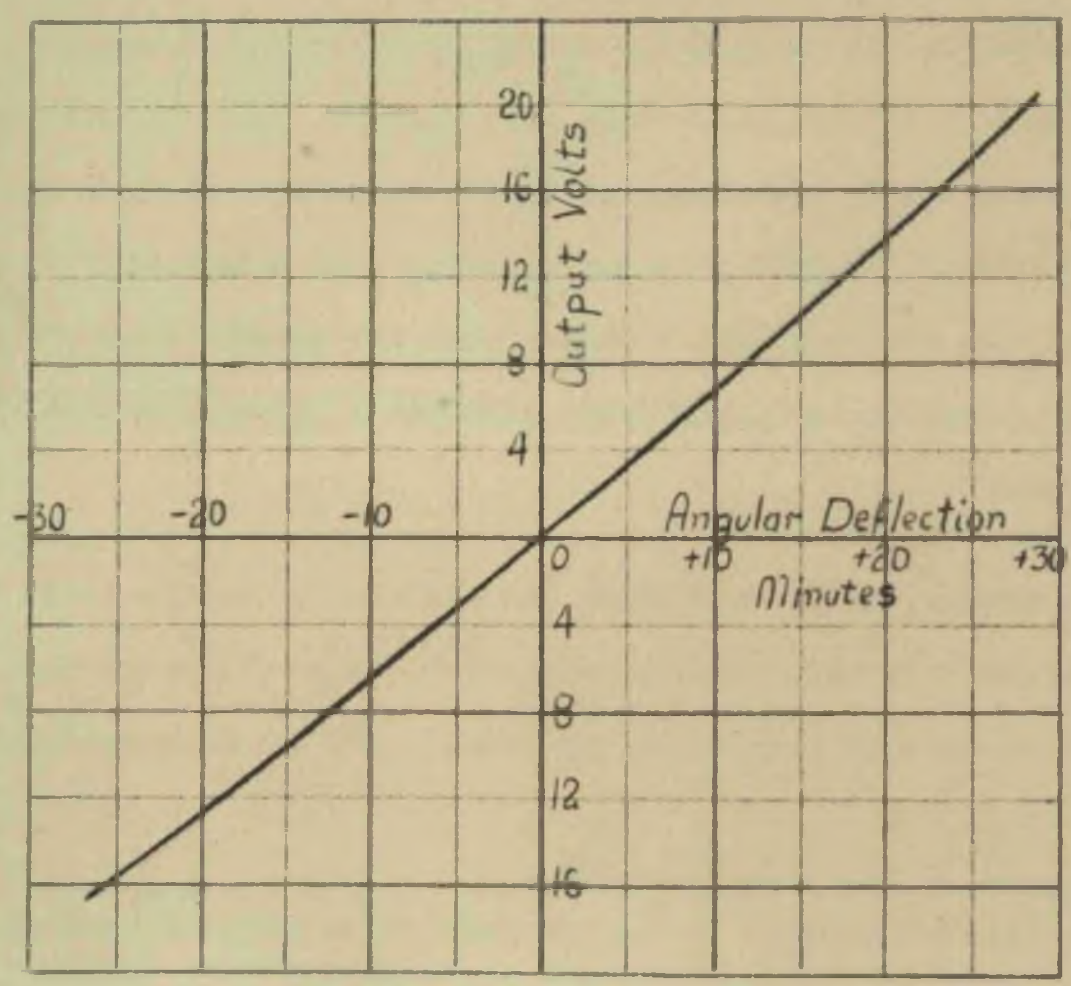

Fig. 19. Characteristic for R.C.A. Transducer Triode with 75-kΩ anode-load resistance and Vg = 0.

The effect is most serious with a large-displacement, low-frequency vibration and the cable connecting the moving pick-up head to a stationary vibrometer. In these circumstances, a battery-operated cathode-follower unit close to, or as part of, the pick-up will act as an impedance transformer and effectively eliminate the trouble. This unit must not be microphonic in any respect, as it is being subjected to the vibration.

In the ship-model tests the effect was noticed but, after some investigation, it was decided that the signal introduced by cable noise was negligibly small compared to the crystal output.

4.7 Triode Transducer Valve

The construction of this device is shown in Fig. 18. The anode is continued through the centre of a thin metal diaphragm and displacement of the external shaft alters the grid-anode spacing which, in turn, changes the anode current. A typical characteristic is given in Fig. 19, from which it will be seen that the sensitivity is about 40V/degree of angular deflection. With a weight of 1/16 oz and an overall length of 1 1/4" this device has many applications. The natural frequency of the anode-diaphragm spring-mass system is 12 kc/s.

A useful accelerometer of low mass was constructed round this valve, Plate I(a). The resonant frequency was reduced to about 400 c/s by attaching a small extension piece to the anode shaft. The valve was contained along with its anode load resistor in a small brass box, which was filled with oil to give the requisite damping.

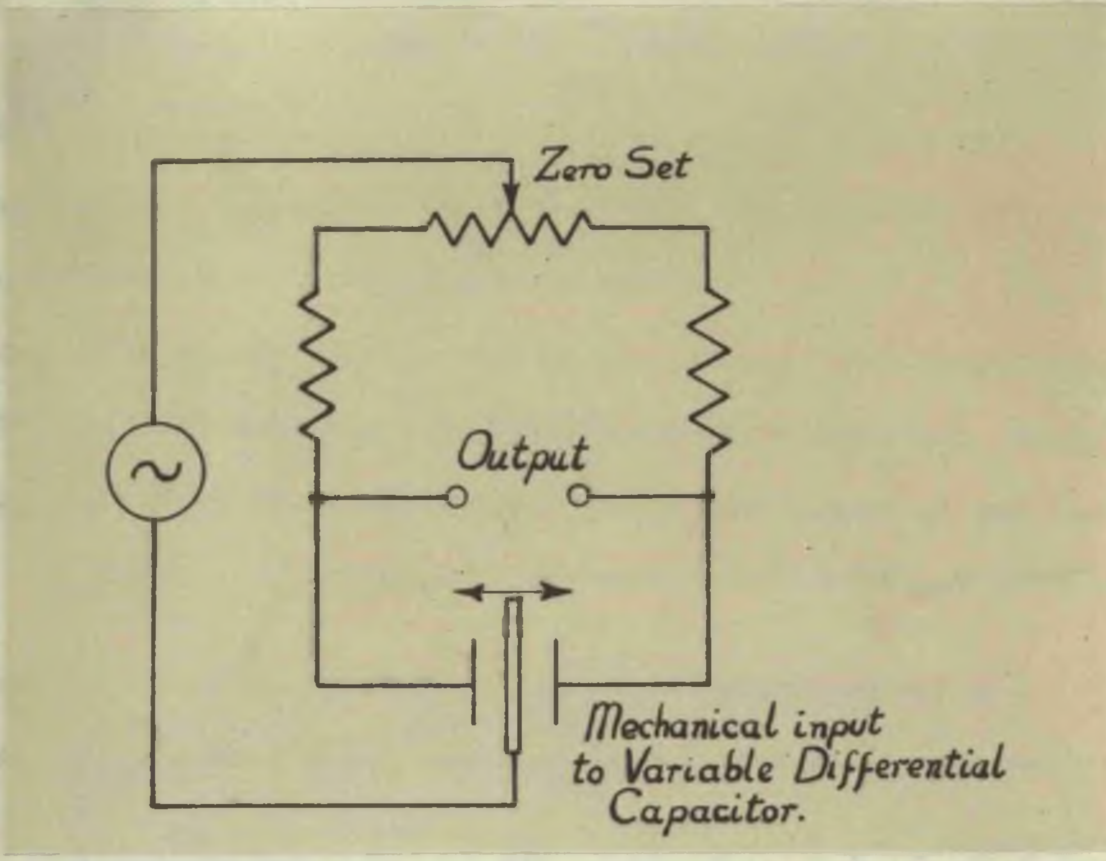

Fig. 20. Simple arrangement for a variable-capacitance transducer.

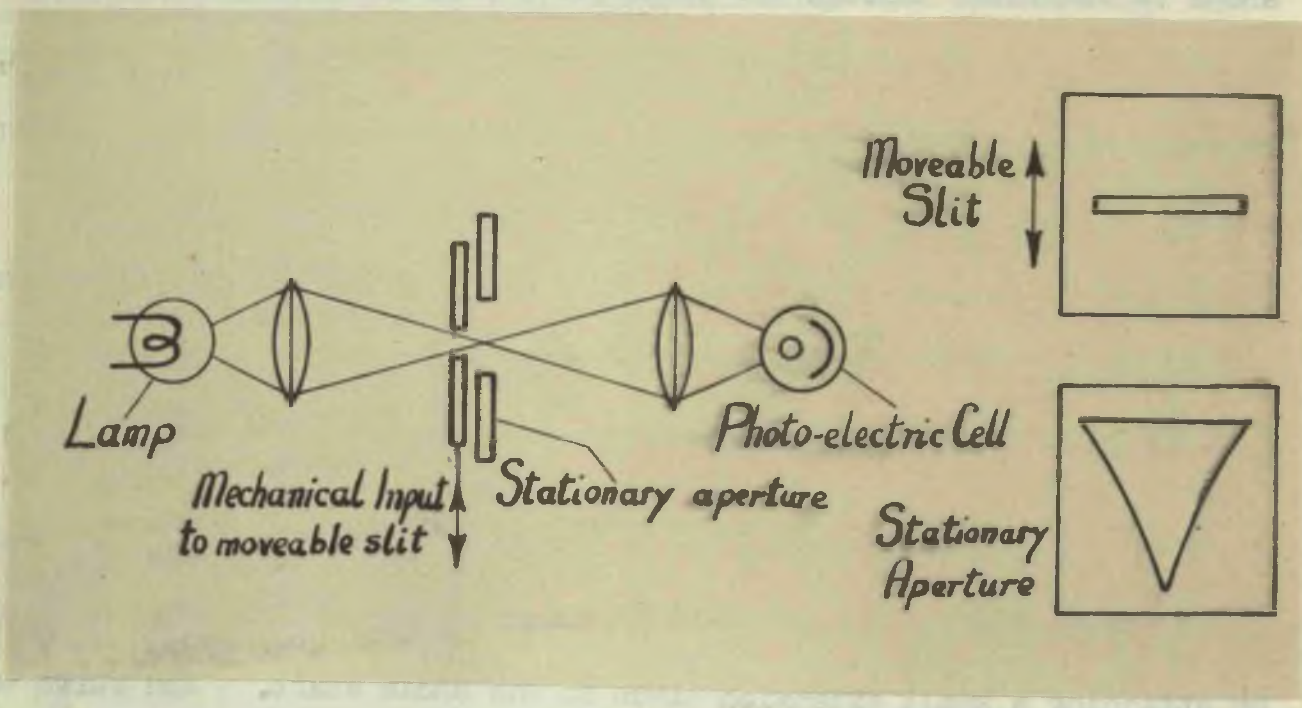

Fig. 21. Photo-electric cell in a simple form of transducer using a modulated light beam.

The output from the pick-up was relatively high, but it tended to be noisy, possibly due to the whole valve being vibrated,and no extensive practical use was made of it. Where the shell of the valve can be attached to a fixed reference and the vibrating body connected to the anode shaft through a rod, the output will be about 20V/.001", which would make a very sensitive and compact transducer.

4.8 Miscellaneous

There are many more possible devices for converting a mechanical to an electrical signal. Some of these are mentioned below.

4.8.1 Change in resistance of a semi-conductor when strained.

The effect is generally greater than that obtained from the metallic strain gauge, but temperature and voltage effects tend to be serious and limit its practical use.

4.8.2 Capacitance bridge, Fig. 20.

Similar in use and application to the inductance bridge. By using high carrier frequencies very compact units may be designed.

4.8.3 Optical methods.

Fig. 21 indicates how a mechanical displacement may be used to modulate the light falling on a photo-electric cell and so produce a voltage proportional to displacement. The shape of the fixed aperture may be modified to compensate for non-linearity of the cell or amplifier response, or to give any required transfer function between input and output signals.

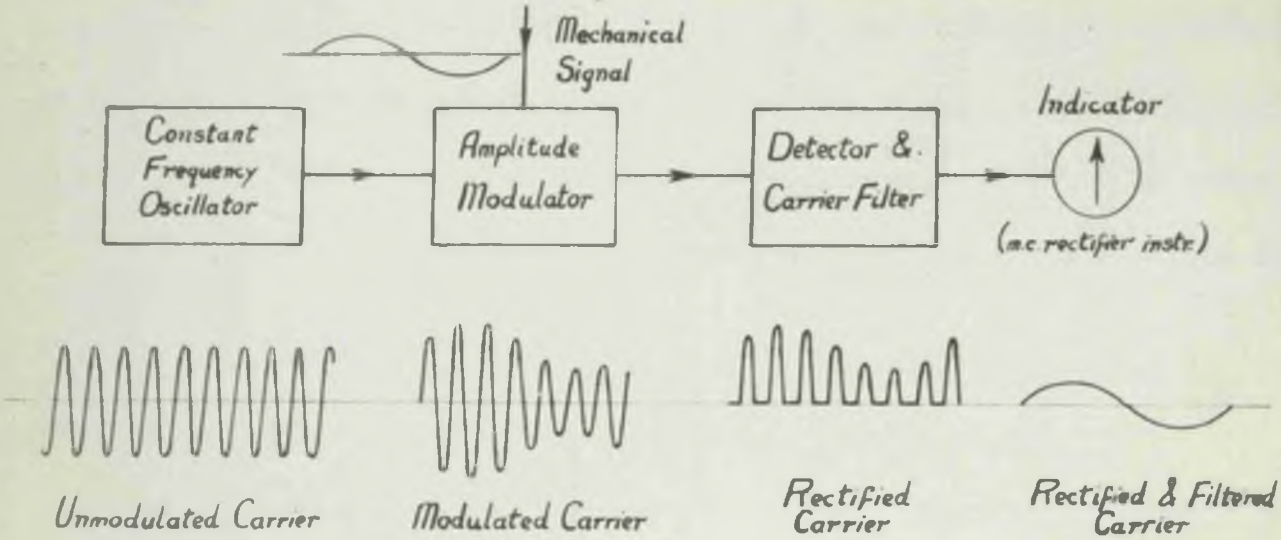

Fig. 22. Block diagram of an elementary amplitude-modulation system.

4.9 Carrier Systems

Where the vibration frequency spectrum extends down to 0 c/s or very low frequencies, it may be convenient or necessary to employ a carrier system.

4.9.1 Amplitude Modulation

The inductance bridge, Fig. 13, is normally balanced so that there is no output when $\Delta L = 0$, the supply or carrier frequency is thus suppressed. If the bridge is now appreciably unbalanced by altering one of the resistive arms, there will be an output at the carrier frequency when $\Delta L = 0$ and the amplitude of this output will vary with both the magnitude and the sign of ΔL, that is, the output now responds to the sense of the displacement. The block diagram, Fig. 22, indicates the essential components of an amplitude modulation system.

With a carrier frequency f_c and a vibration frequency f_s, the waveform of the modulated carrier is given by an expression of the form,

$$v = V\,(1 + k \sin 2\pi\, f_s t) \sin 2\pi\, f_c t \qquad (55)$$

Analysis of this expression shows that the required frequency band for the amplifier is twice the maximum vibration frequency centred on the carrier frequency. This amplifier is often easier to design and more reliable than a stable d.c. amplifier of the same gain. There is the additional complication of the demodulation circuit.

Amplitude modulation is also applicable to resistive strain gauge and capacitance bridges as well as valve and photo-cell transducers.

The carrier frequency is a matter of convenience but generally it should be several times the highest vibration frequency to simplify the demodulation filter circuit. Too high a carrier frequency may introduce complications, such as the effect of circuit and component reactances.

4.9.2 Frequency Modulation

In an alternative system, the mechanical signal varies one of the elements in an oscillator circuit, so that the oscillator frequency changes in sympathy with the signal. As the range of frequency is usually only a very small fraction of the carrier frequency, special circuits have been devised to detect the modulation. Fig. 23 shows an elementary system. The two halves of the detector are tuned to frequencies above and below the unmodulated carrier frequency and their outputs are subtracted electrically. The result is an output which is linear over a reasonably wide frequency range. Two of the more obvious advantages of the system are that zero or very low-frequency modulation may be used and the discriminator and presentation unit may be situated at a distance from the pick-up head without any loss of accuracy.

4.10 Integration and Differentiation

The electrical output from a vibration pick-up may be proportional to x, $\dot{x}$, $\ddot{x}$ or $\dddot{x}$, depending on whether the spring-mass system operates above or below its natural frequency and on the type of transducer. It is convenient, therefore, to include in an electronic vibration meter, circuits that will operate on the electrical signal representing the

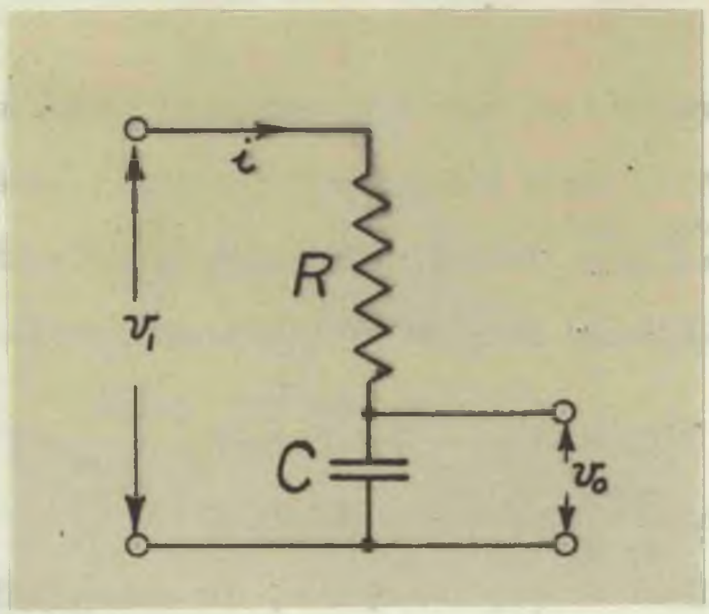

Fig. 24. Simple integrating circuit.

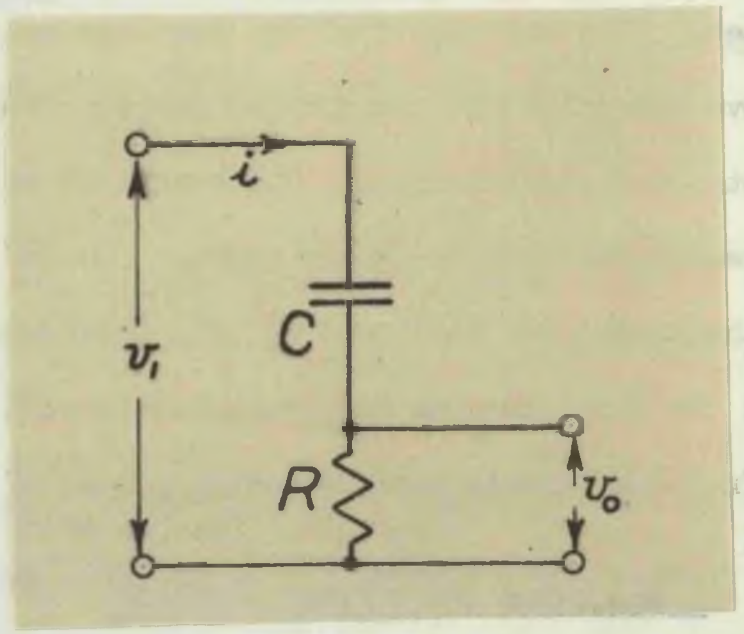

Fig. 25. Simple differentiating circuit.

vibration to give the required parameter. Most commercial units give a choice of parameter by operation of a selector switch.

4.10.1 Integration. Fig. 24

$$v_o = \frac{1}{C}\int i \, . \, dt = \frac{1}{C}\int \frac{v_1 - v_o}{R} \, . \, dt$$

$$= \frac{1}{CR}\int v_1 \, dt - \frac{1}{CR}\int v_o \, . \, dt$$

$$\doteqdot \frac{1}{CR}\int v_1 \, dt \qquad (56)$$

provided that the second term is small. This occurs when $R \gg \frac{1}{\omega C}$ for all frequencies of interest. A useful criterion is that 1% accuracy is obtainable with a time constant of 1 sec at a frequency of 50 c/s.

At low frequencies accurate integration with a simple circuit may be difficult and it is then common to employ a "Miller Integrator". This is an amplifier with 180° phase shift and capacitance feedback between output and input. It has a number of advantages over the simple circuit given above[12].

4.10.2 Differentiation. Fig. 25

If $v_o \ll v_1$, the charge on the capacitor $q \doteqdot Cv_1$ and $i = dq/dt \doteqdot C\frac{dv_1}{dt}$

$$\therefore \; v_o = iR \doteqdot CR\frac{dv_1}{dt} \qquad (57)$$

provided $R \ll \frac{1}{\omega C}$

It is apparent that, for either integration or differentiation, there must be considerable attenuation of the signal if accuracy is to be attained. An extra stage of amplification is normally necessary to compensate for this loss.

5. PRESENTATION

The method of presentation depends on the equipment available and the type of test being carried out. For laboratory work, where only one or two detectors are being employed and note is being taken of the effect of change in the various experimental parameters, a direct indication on a dial instrument has much to recommend it. This can be usefully supplemented by visual presentation of the vibration waveform on a cathode-ray tube, so that immediate note may be taken of peculiarities in this waveform, such as harmonics or the presence of vibrations of other frequencies.

For the simultaneous operation of a number of pick-ups, this method is laborious and can lead to error, due to changing conditions while the readings of the several detectors are being noted. Here, some type of recording is desirable. It may take one of several forms:

Simultaneous photography of a number of instrument dials or cathode-ray tube screens.

Multiple-channel oscillograph, employing taut-suspension reflecting galvanometer movements and a moving photographic film to record the movements of the light beams.

Multiple-pen recorder - usually of the moving-coil type, which limits the upper-frequency response to a maximum of about 100 c/s.

Magnetic-tape recorder. This does not seem to have been developed commercially in this country to the extent of being sufficiently accurate for measurement purposes.

The multiple-pen recorder has much in its favour where the frequencies of interest are within its range, as the records are immediately available.

In the ship-model tests no recording equipment was available and all measurements were presented on a rectifier moving-coil instrument. As the vibrations were substantially sinusoidal, this gave accurate readings. Where the signal waveform is non-sinusoidal, errors which may be large, are introduced[13]. These errors are also a function of the phase relationships between the components of a complex wave, so that calibration for non-sinusoidal waveforms is virtually impossible. However, by introducing a tuneable selective amplifier before the instrument, the magnitudes of the several components may be measured individually.

The errors mentioned in the previous paragraph arise because the instrument deflection is proportional to the rectified-mean value of the signal while the scale is calibrated in r.m.s. values for a sinusoidal input. Where accurate r.m.s. values are required in the presence of harmonics, some form of thermal instrument is recommended. The relatively long time constant of thermo-junction instruments is also an advantage for obtaining a steady reading.

Where the vibration waveform departs substantially from the sinusoidal form and peak values are of interest for determining peak deflections and forces, a valve voltmeter with a suitable R-C time constant is desirable.

From the above, it can be seen that the pointer instrument can be arranged to indicate the r.m.s., mean or peak value of a waveform and this information is often adequate. Where the waveform is of interest, the cathode-ray oscilloscope is the most versatile instrument, but its accuracy may be limited by a number of factors, namely:

(a) non-linearity of the amplifiers over the frequency range;

(b) non-linearity of the time-base;

(c) non-proportionality of the spot deflection to the applied voltage;

(d) dependence of spot brilliancy on writing speed, making photographic recording and analysis difficult with marked non-sinusoidal waveforms;

(e) width of trace or spot size;

(f) parallax and "sine-tangent" errors. These may be accounted for in (c);

(g) camera errors due to poor lenses and distortion of the recording film or paper during the development process.

By careful design and manufacture and by taking suitable precautions, good accuracy can be attained in spite of the above sources of error.

Photographic recording has the disadvantage that the record is not available until the film is developed, so that in the event of the record being unsatisfactory, considerable trouble and expense may be involved in repeating the test. If the special materials and cameras, advertised in the U.S.A. literature, for giving a useful record in less than a minute after exposure are really satisfactory and become available here, they will provide a marked advance in photographic recording technique.

Magnetic-tape recording theoretically should offer many advantages and many claims are made for special instruments. However, it appears that even in the best of tapes, the characteristics can vary by about 10% so that a direct amplitude recording, although relatively simple, is not suitable for accurate work. Various frequency-modulation systems have been reported, the best of these has a tape speed of 60 in/sec and a nominal carrier frequency of 45 kc/s. This unit is claimed to be capable of handling satisfactorily a signal range of 0 to 10 kc/s. Elaborate methods of controlling the tape speed have been suggested and described, but the one which appears to offer the best all-round performance is the recording of a reference frequency on a second channel of the same tape as the signal. The mechanical problems of speed control appear to be more intractable.

6. PRODUCTION OF VIBRATION

Vibrations may be induced in a system by a number of methods. These are considered briefly.

6.1.1 Shock

The system is hit a blow, preferably of known and repeatable force,and the resulting vibration is recorded. The analysis of this record is mathematically interesting and can yield much information, but the method was never seriously considered in this project.

6.1.2 Mechanical

The system can be excited by a cam or eccentric drive which moves the system relative to the ground or a large reference mass. This method is not easily applicable to a freely supported system. An alternative mechanical method is to utilise the centrifugal force of a rotating out-of-balance mass. This method was adopted and is discussed further in Section 6.2.

6.1.3 Electrical

Certain of the transducers discussed in Section 4 exhibit a reciprocal effect in that an electrical input produces a corresponding mechanical force output. The most popular is the moving-coil exciter, which is available commercially in all sizes up to large units capable of force outputs of 10,000 lb.wt. It has many advantages, but was not considered suitable for ship-model work. Some doubt has since been felt

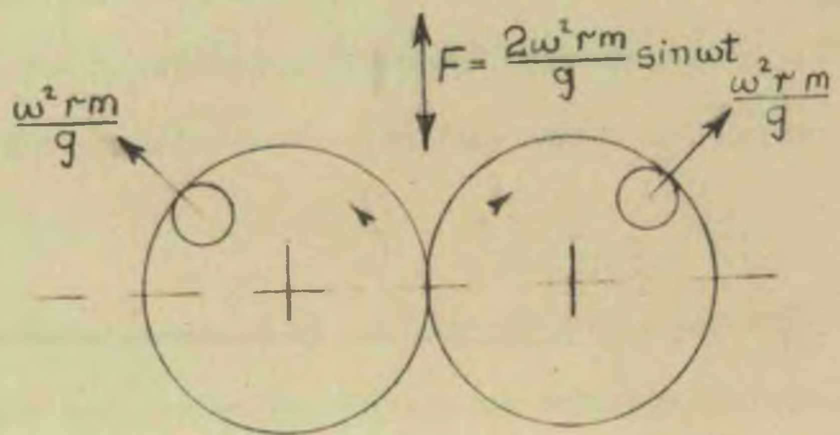

Fig. 26. Two-mass out-of-balance vibration exciter.

about this decision, as the mechanical disadvantages of the scheme applied to the free-free condition may be outweighed by the electrical convenience. At the lower frequencies, some trouble may be experienced in designing a suitable transformer for matching the amplifier output to the moving coil.

Other electrical methods include the moving-iron electromagnetic exciter, which has been used for the tests on wax bars (Section 13), as well as electrostatic, piezo-electric and magneto-strictive effects. The last two are used extensively at the higher frequencies.

6.2 Rotating Out-of-Balance Exciter

If a carefully balanced wheel has a mass m attached to it at radius r and the wheel is rotated at $f = \omega/2\pi$ revolutions per second, there is a centrifugal force

$$F = \omega^2 r \frac{m}{g} \tag{58}$$

The direction of the force is radially outward from the axle, so that there is a considerable possibility of exciting several modes of vibration simultaneously with this simple system.

If two carefully balanced gear wheels with attached masses m are arranged as in Fig. 26, the horizontal components of the centrifugal forces balance out while the vertical components add to give a resultant

$$F = 2\omega^2 r \frac{m}{g} \sin \omega t \tag{59}$$

By displacing one of the masses 180°, the vertical forces cancel and a horizontal sinusoidal force plus a couple are produced.

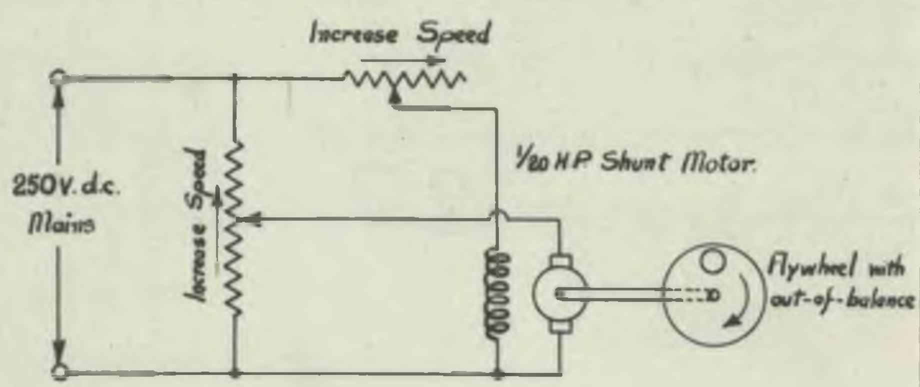

Fig. 27. Exciter Mark I.

Other arrangements of two, three or four out-of-balance weights can be designed, but they all involve more complication in gearing which outweighs their other advantages, particularly for high-frequency operation.

6.3 Development of Excitation System

Initially, little was known about the precise requirements and a simple exciter, Fig. 27, consisting of a $\frac{1}{20}$-h.p. shunt motor with a disc flywheel and a single out-of-balance mass was mounted in a wax model. This unit had a maximum speed of 3000 rev/min, speeds being measured by an electronic stroboscope. With the model mounted on modified wooden knife-edges, resonances were readily detected by touch. Some attempt at quantitative measurements was made with the seismic detector mentioned in Section 4.5, but the results were sufficiently inaccurate to have little value.

From these early tests, it was apparent that:

Comparatively little power was required to generate amplitudes detectable by accelerometers available commercially.

The resonances were sufficiently sharp to be difficult to hold by the simple slide-wire resistance speed controls.

The simultaneous existence of complex vibration modes - vertical, horizontal and torsional - was caused by the simple rotating out-of-balance mass.

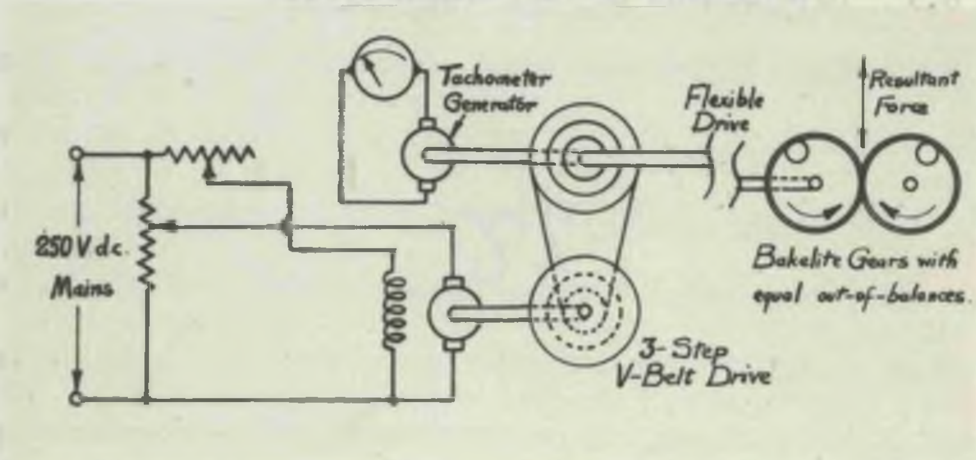

Fig. 28. Exciter Mark II.

Both the exciter and detector were too heavy.

The speed range of the motor was insufficient.

As a result of experience with this exciter, an improved arrangement, Fig. 28, consisting of a relatively large d.c. shunt motor, "shore-based", driving an exciter through a flexible speedometer cable was constructed. The motor was connected to the cable through a three-step V-belt pulley drive, so that a wide range of exciter speed was available with a comparatively small range of motor speed. This, along with the large inertia of the motor armature led to greater ease of speed control. The exciter speed was measured on a 10-inch "Cirscale" voltmeter, supplied from a d.c. tachometer generator driven from the exciter shaft. The exciter proper consisted of two accurately machined bakelized-fabric gears with provision for attaching out-of-balance masses. The complete arrangement of this equipment, as used at Leven Shipyard, is shown in Plate 2. In the experiment shown, an aluminium-alloy model of rectangular cross-section is being tested.

The Mark II unit was a great improvement on the original Mark I, but it was still difficult to hold a resonant speed sufficiently long for the vibration amplitude to reach its final value and an accurate reading to be obtained from the vibrometer. It was also impossible to plot amplitude/frequency curves near the resonant frequencies. The accomplishment of accurate speed control was now the major outstanding problem of the exciter design.

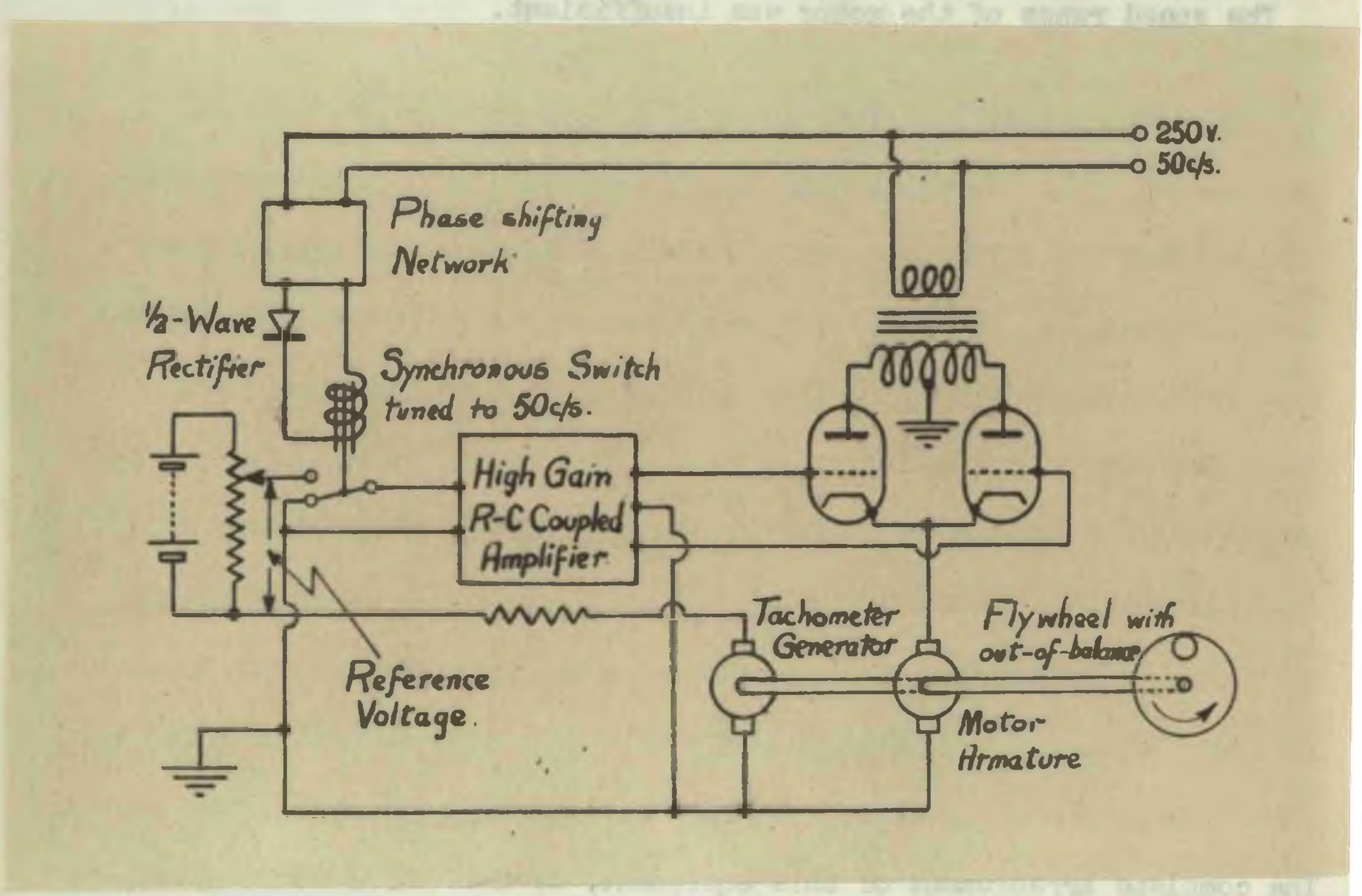

Fig. 29. Block diagram of control system for exciter Mark III.

In order to expedite the programme, a Mark III exciter with electronic speed control was rapidly constructed. The functional diagram, Fig. 29, shows the system employed. The output from the tachometer generator is compared with a reference voltage derived from a dry battery and a potential divider network. The difference voltage is chopped by a vibrator tuned to 50 c/s and fed to a high-gain amplifier which, in turn, controls the motor armature current.

The resulting speed control was fairly effective over the range 300 to 6000 rev/min but, owing to the small torque of the motor, only small out-of-balance masses could be employed and the usefulness of the unit was limited.

Plate 3 shows the general appearance of the unit and the complete circuit is given in Circuit 1.

6.4 Final Design of Excitation System

With the above experience as a guide, a Mark IV exciter using a Velodyne Motor-Generator, Type 74, was designed and constructed.

The weight of this machine, $7\frac{1}{2}$ lb, is too great for it to be mounted on the model, so the exciter is driven by a short length of torsionally stiff, flexible control cable.

The exciter consists of two precision bronze gears, 3 inches in diameter, mounted on ball bearings and running in a totally enclosed oil bath. By adopting a light but stiff brazed construction, the

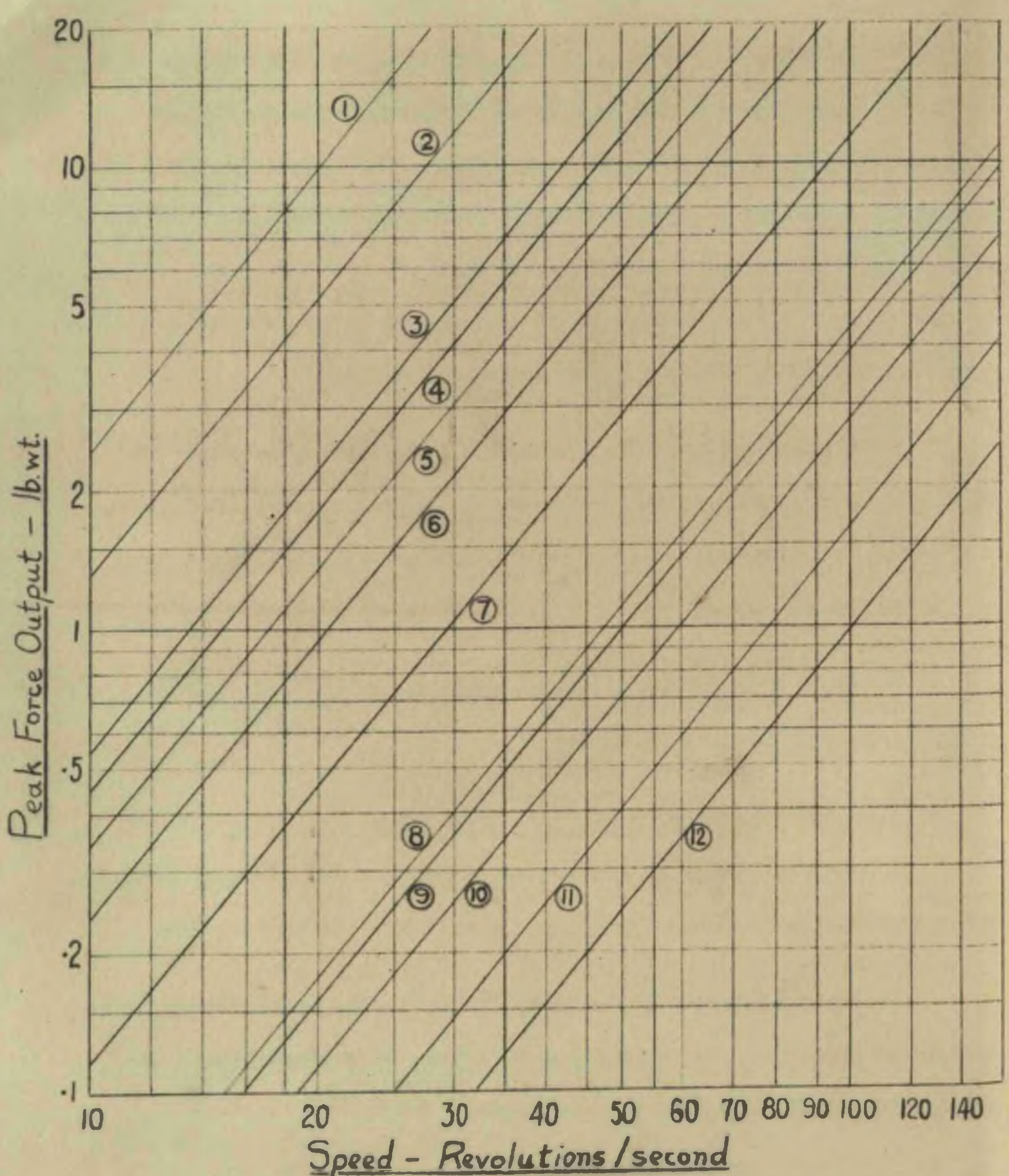

Fig. 30. Calibration curves for out-of-balance weights for exciter Mark IV.

weight of the exciter with its mounting fitted in the model is less than $3\frac{1}{2}$ lb. This construction is seen in Plate 4. A set of twelve pairs of accurately matched weights is available, so that a wide range of force outputs over the required speed range can be obtained. The calibration curves for these weights are given in Fig. 30. The larger weights can produce forces sufficiently great to damage the ball races at quite modest speeds. The maximum permissible force is a function of speed and the expectation of life for the ball races. No calculation has been made for a suitable maximum force output, but an arbitrary limit of 15 lb.wt. has proved to be sufficient to excite vibrations of sufficient magnitude under all conditions of model testing and the life of the ball races has been satisfactory. After more than 1000 hours of running time, a small amount of noise is detectable.

6.4.1 Velodyne Motor-Generator Control System

The principle of operation is shown in Fig. 31. The motor armature is supplied with an effectively constant current of 5A through a series resistance from the d.c. mains. The back e.m.f. (4V per 1000 rev/min) has virtually no effect on the armature current, so that the unit produces a constant indicated torque over the whole range of speed from 0 to 10,000 rev/min. This torque is of the order of 1500 gm cm or 20 oz in but, due to friction and windage losses, is less at high speeds.

Mechanically coupled to the motor is a d.c. generator, whose output of 40V/1000 rev/min is accurately proportional to speed. The

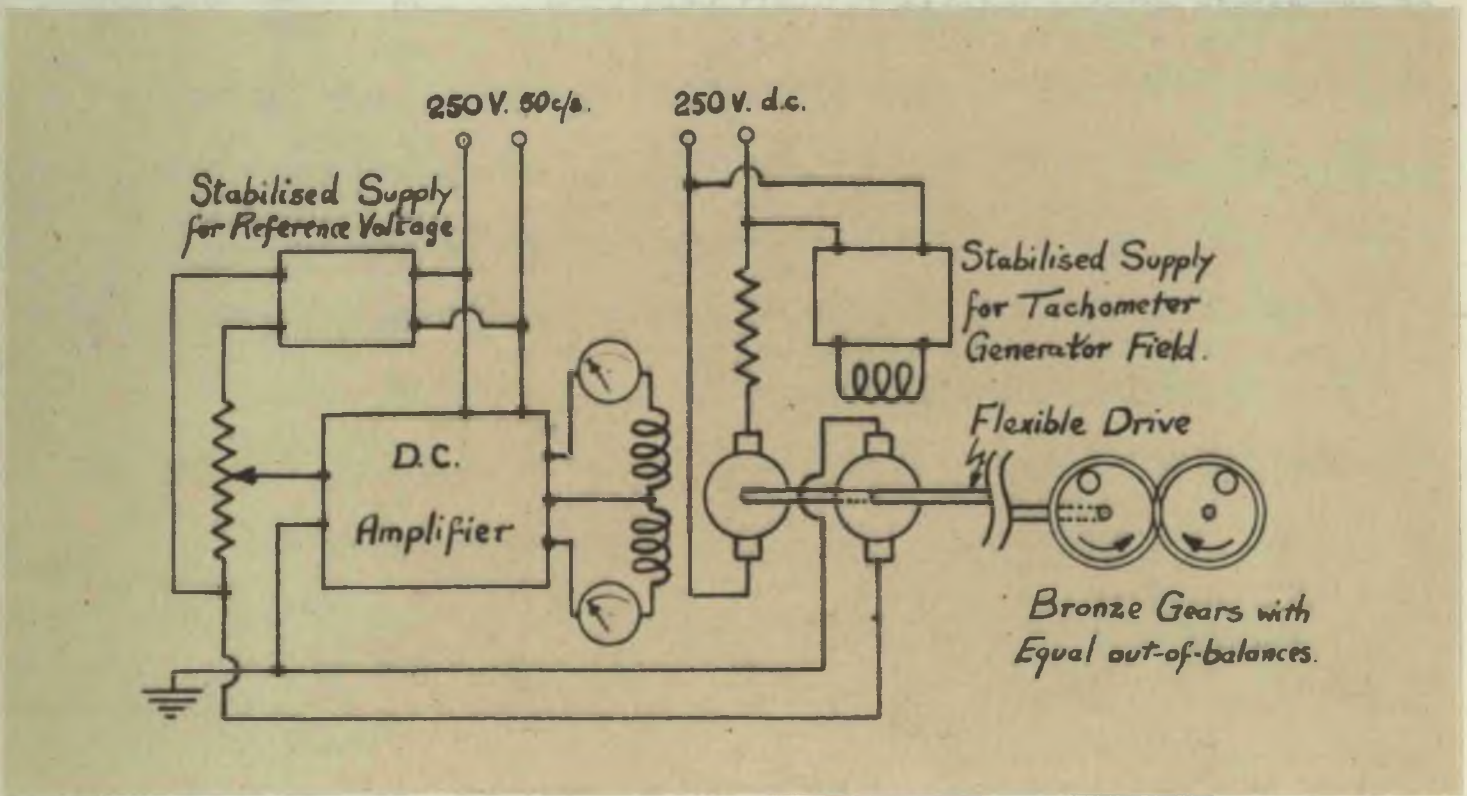

Fig. 31. Block diagram of control system for exciter Mark IV.

d.c. supply for the generator field is electronically stabilised, so that the output is substantially independent of mains variations. The generator output is connected in series with an electronically stabilised reference voltage, which is variable from zero to 300V. The difference, or error voltage, is amplified in a d.c. balanced amplifier, whose output is applied to the motor field coils. Full field current is obtained from an input of 120mV, corresponding approximately to 5 rev/min, Fig. 32. Thus, full torque is developed with an error input of 5 rev/min, giving a theoretical accuracy of speed control of 0.17% at 3000 rev/min with full range of torque variation.

The inertia of the motor generator alone is about 8 oz.in.sec^2 and the time constants of the amplifier and feedback circuits are low, so that very rapid rates of speed change are possible - up to 9000 rev/min/sec - and the effect of transient loading has a very short duration - about 50 m sec.

When used with the out-of-balance exciter, the inertia of the system is increased by about 50% and an additional complication is introduced by the flexible shaft coupling between the velodyne and the exciter. With a stiffness of about 60 oz.in/radian for a length of 14 in, the calculated resonant frequency for the system is about 8 c/s. In practice, with normal rates of change of speed and due to the damping, no trouble has been experienced with this form of coupling.

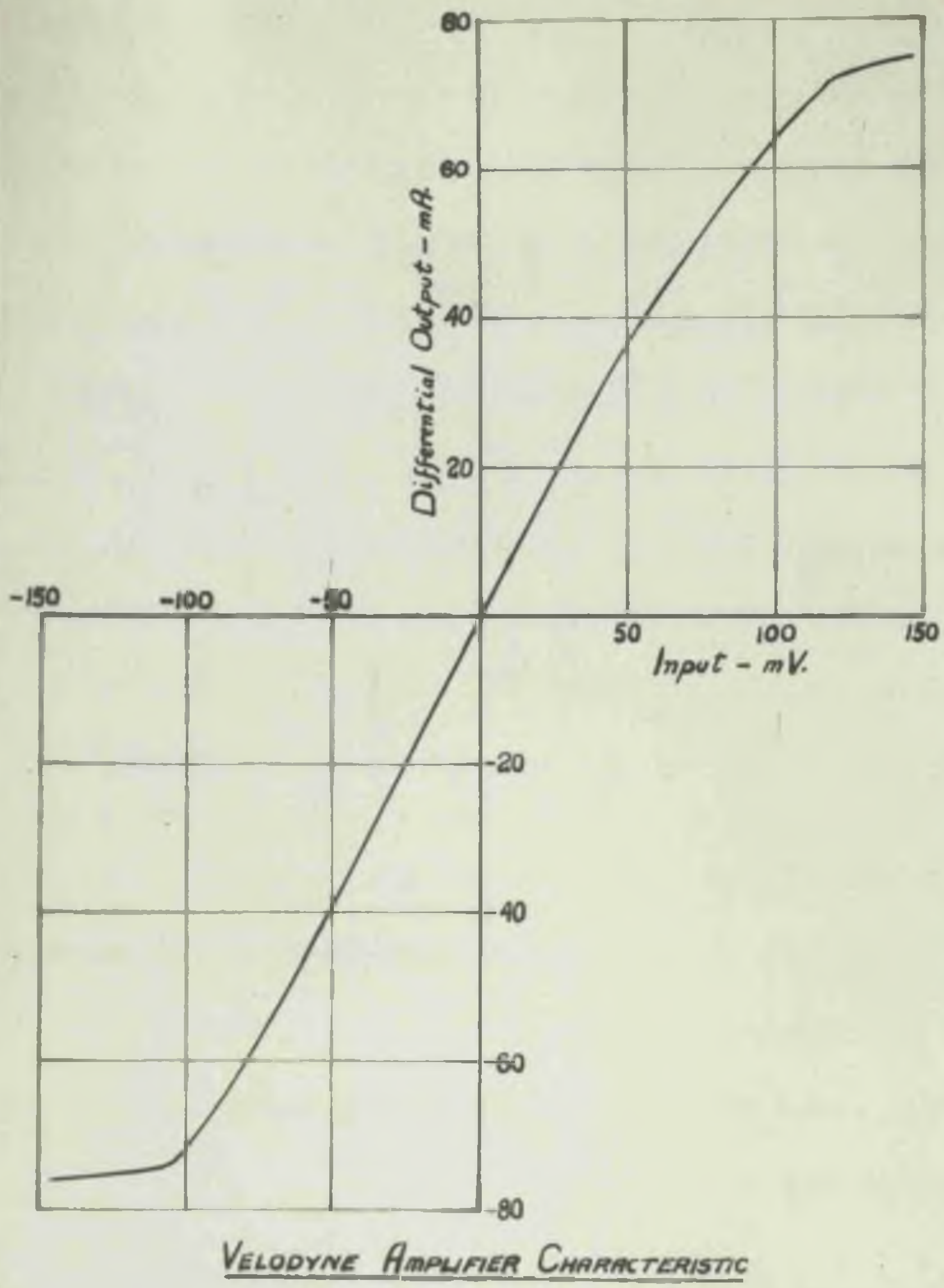

Fig. 32. Response curve for amplifier used in exciter Mark IV.

The associated control and amplifier units have been designed with modest rating of all components, so that they have functioned without failure for about 2000 hours of running time. The circuit of the d.c. amplifier is substantially similar to that described by Williams and Uttley[14].

Complete details of all the units are given in Circuits 2, 3 and 4 and they are illustrated in Plates 5 and 6.

6.4.2 Measured Accuracy of Velodyne Speed Control

Owing to the lack of precision speed-measuring equipment at Glasgow, tests on this unit were limited to some simple observations. For example, with the shaft of the motor apparently stationary, when illuminated by an electronic flashing stroboscope unit, the change in speed from zero to full-load torque was estimated at about 10 rev/min. Theoretically, it should be nearer 4 rev/min. Later, tests at the National Physical Laboratory indicated that constancy of speed in the region of 1 to 2 rev/min was obtainable over a wide range of speeds with a fair range of torque. It was also found that the selected speed remained constant over a long period of time. Some tests to determine the calibration accuracy of the unit showed that, once set up, the actual speed was within 0.5% of the indicated speed. As the resistors making up the reference voltage potential divider were all wire wound and adjusted to better than 0.1%, the difference is probably due to neon variations and the effect of commutator-brush contact potential in the generator.

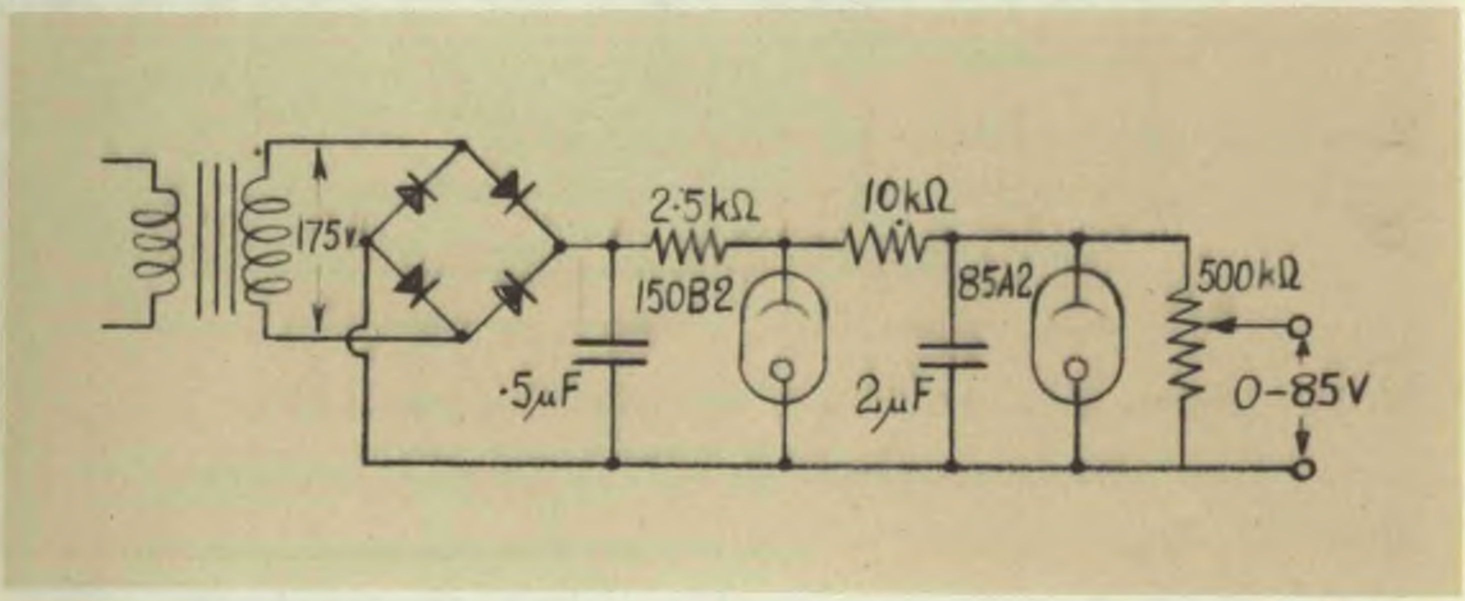

Fig. 33. Circuit for a double-stabilized reference voltage system.

6.4.3 Close Speed Control

The speed of a motor may be precisely controlled in speed under certain conditions. For example, if the speed is continuously monitored by some form of digital counter and the error used as a feedback signal, the resulting control should be absolute within the accuracy of the counting system. However, if a generator output is compared with a reference voltage, then a number of errors may enter into the system. For example, a typical sintered alloy permanent magnet has a variation in flux of 0.03% /°C and this may be sufficient to rule out the use of a permanent magnet generator for high stability use. If a d.c. field is used, it is commonly excited from a rectified supply, which is stabilised. A second stabilised supply is used for the reference voltage. In order to attain precise speed control, certain precautions must be taken. Some quoted temperature coefficients are + .07% for dry cells, + .01% for lead acid cells and -.005% for neon stabiliser valves, all values being per °C. Neon stabilisers, therefore, appear most suitable but, in addition, they are subject to the effects of variations in mains voltage. A typical figure is ± .06% change in output for a ± 10% change in mains supply. Therefore, where high stability is of importance, a double stabilised supply, as shown in Fig. 33, is useful[15]. A rough calculation shows that this circuit should give a very satisfactory output. However, it appears from the literature that neon-stabilisers are subject to other deficiencies which make elaborate circuits futile[16].

A new reference source has been suggested by Shields[17]. The saturation voltage (breakdown region) in the reverse characteristic of a silicon diode can be arranged to have a value from a few volts to several hundred volts. The temperature coefficient is given as 0.06V/°C, which does not compare very favourably with existing sources. However, the advantage of low voltages may be considerable.

All precautions for securing a standard reference may be invalidated by commutator-brush contact potentials. Various suggestions have been made to improve conditions in this respect and silver-graphite brushes on a silver-palladium commutator have been claimed to be successful but would doubtless be prohibitively expensive for a machine of the size used.

7. MEASUREMENT OF PHASE

When it became apparent that useful results could be obtained from the model tests, there was a request for a method of measuring the phase relationship between the exciting force and the vibration displacement. The estimation of phase from elliptical patterns on a cathode-ray tube was considered to be too laborious, so an instrument was developed for the purpose.

The measurement of phase is important for two reasons,- firstly, the calculation of power input to the vibrating body and, secondly, the determination of the resonant frequency of a highly-damped structure. The amplitude/frequency response for a highly-damped structure is very flat near the resonance frequency and it may be difficult to determine this frequency with accuracy. On the other hand, the rate of change of phase angle with frequency is greatest near resonance so that phase-measuring technique can lead to a more accurate estimate of the resonant frequency.

7.1 Phase-Measuring Unit

As the experimental technique required a relatively large range of frequency, the measurement of phase relationships offered some difficulty. However, as the waveforms of interest were substantially sinusoidal, the problem was considerably simplified. It was important that there should be more than one channel so that the performance of a complex structure could be evaluated. The basic principles of the unit

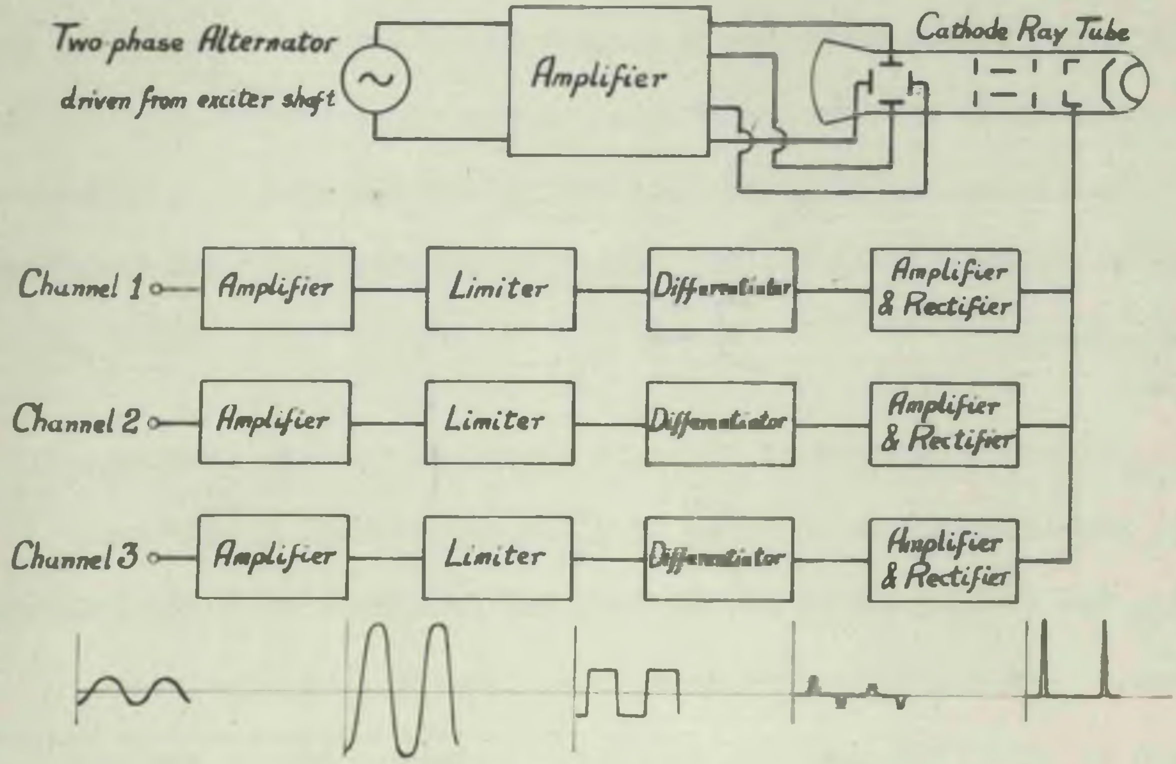

Fig. 34. Block diagram of phase-measuring unit.

as finally constructed are shown in Fig. 34. A two-phase alternator driven from the same shaft as the exciter produces a circular time-base on a cathode-ray tube. There is provision for three signals into the unit,- namely, a reference signal and two others whose phase relative to the reference is to be compared. The technique is that each signal is amplified as necessary, clipped, amplified, clipped again, differentiated and, after further amplification, fed to the grid of the cathode-ray tube to produce a dark or a bright spot, according to polarity, on the circular time-base.

In front of the screen is a cursor made of thick Perspex with a number of concentric circles and a radial line engraved on both sides to eliminate parallax errors. This disc is rotatable in a metal hood which is itself capable of being rotated. Engraved round the hood are 360 degree divisions. The procedure for operation is to centre the time-base and set the hood to 0^o and the radial line on the cursor to the reference spot. The hood and cursor are then turned together until the line is over the spot due to the pick-up signal. The phase angle is read off from the graduated scale. Although the accuracy of reset is better than 1^o, it is probable that the limiting accuracy due to defocus, harmonic content of the vibration waveform, unequal phase shift in the amplifiers and parallax errors, is nearer $\pm 4^o$. In addition, if there is fluctuation in the vibration for any reason, this will result in some loss of discrimination. Under normal conditions with good speed control, there is no noticeable fluctuation.

Although in no sense a precision instrument, the unit appears to be sufficiently accurate for the purpose for which it was designed.

7.1.1 Details of Construction

The whole unit, Plate 7, is contained in a box 16 3/4" x 7 3/4" x 8", and, as can be seen from the photographs, the available volume has been effectively filled. External to the unit is the two-phase generator, which is a modified 2" Magslip Transmitter, Type E-3-A/1. This generator is flexibly coupled to the Velodyne Motor Generator and is connected electrically to the phase-measuring unit by an eight-core cable and plug-and-socket system. The connections for this portion of the unit are given in Circuit 5.

The output from the magslip is filtered to remove any unwanted ripple and to reduce the harmonic content, Circuit 5. The output from the filter is amplified, Circuit 6, and fed to the X and Y deflecting plates of the cathode-ray tube. The resulting trace is effectively a circle over the frequency range of 10 to 200 c/s. At the higher frequencies, the magslip unit tends to run a little warm but it has operated continuously for long periods at speeds around 5000 rev/min (83 c/s) apparently without damage. Due to the nature of the filter network, the diameter of the circular trace is not proportional to speed and, over a reasonable speed range, the magslip excitation may be left untouched without the diameter of the trace changing sufficiently to cause inaccuracy.

The gain of each of the three separate signal channels is adjustable and a choice of three differentiating networks depending on the frequency is available, Circuit 7. After differentiation, the pulses are fed to cathode-follower stages and thereafter combined in a common pulse amplifier, Circuit 8. The output may be taken from either the anode or cathode of the last stage, giving blanking or brightening signals respectively.

The power supplies, cathode-ray tube network and shift circuits are conventional, Circuit 9.

An earlier design of the unit with a different type of alternator, using four telephone headpiece inserts and an eccentric rotating iron armature, was used for a short time but difficulty was experienced, due to the relatively small output of the generator and due to hum and stray signals. It will be seen that the later circuits are thoroughly smoothed and decoupled. This has proved most effective in giving a "clean" picture.

Initial tests on the magslip unit showed that it was necessary to keep the excitation current small in order to maintain a good sinusoidal output waveform. The excitation current is derived from a small power supply which is well smoothed and the maximum current is about 30 mA, which is sufficient to give a 1 3/4" diameter trace at 10 c/s.

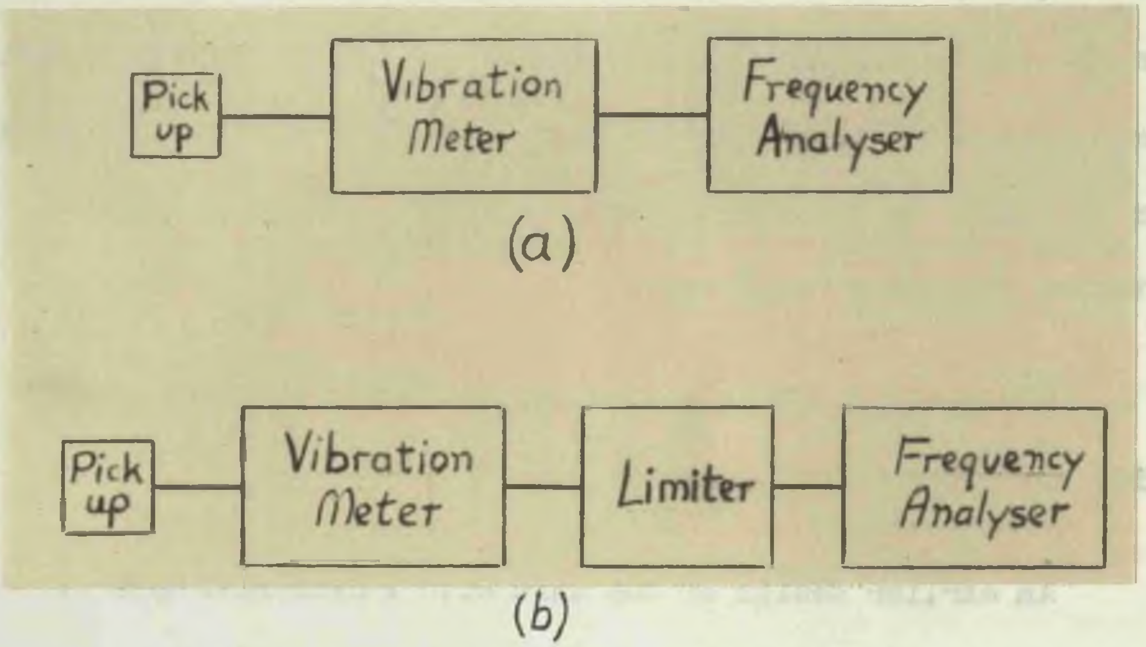

Fig. 35. (a) Block diagram of arrangement for measuring vibration amplitude and frequency;

(b) Measuring system using a limiter.

8. LIMITER

A common arrangement for measuring vibrations is shown in Fig. 35(a). If the vibration amplitude is fluctuating, both the vibration and frequency meter readings will fluctuate and it is difficult to estimate the vibration amplitude and generally impossible to determine the frequency with any degree of accuracy.

Where the fluctuation is fairly rapid, an improvement in the amplitude measurement is obtained by shunting the indicating instrument by a large capacitor. In the case of the frequency meter, the improvement due to this technique is not so marked, possibly due to the effect of the frequency-selective circuits on an input whose frequency and/or amplitude is changing irregularly.

The effect was serious on some of the earlier ship-model experiments and a substantial improvement resulted from the insertion of a limiter unit between the vibration meter and the frequency analyser, Fig. 35(b). If the fluctuations are due to varying frequency, then the limiter does not stabilise the analyser readings.

Initially, a battery-operated limiter was used but later a more versatile mains-operated, two-channel unit, capable of dealing with the whole range of vibration frequencies was constructed. Details of this unit are shown in Plate 8 and Circuit 10. In

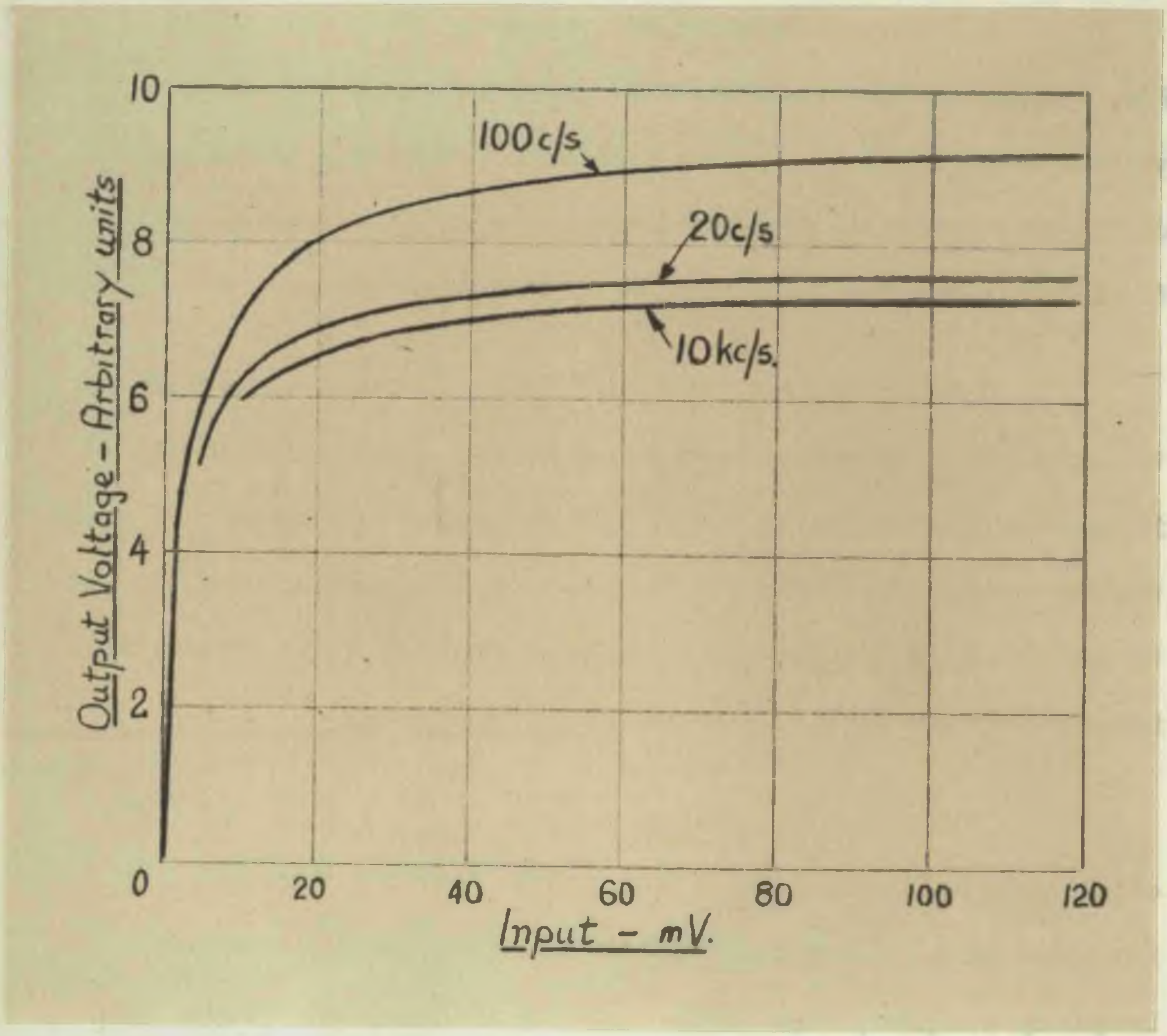

Fig. 36. Response curves for limiter at 20, 100 and 10,000 c/s.

operation, the gain of the amplifier is increased until the instrument on the panel indicates that limiting action is taking place. An accurate reading of the fundamental frequency may then be obtained. As the vibration waveform is of necessity distorted, no evaluation of the harmonic content is obtainable. The response of the unit is shown in Fig. 36.

The limiter was found to be unnecessary in the later tests, when the Mark IV exciter was used and the model was isolated by the rubber support system.

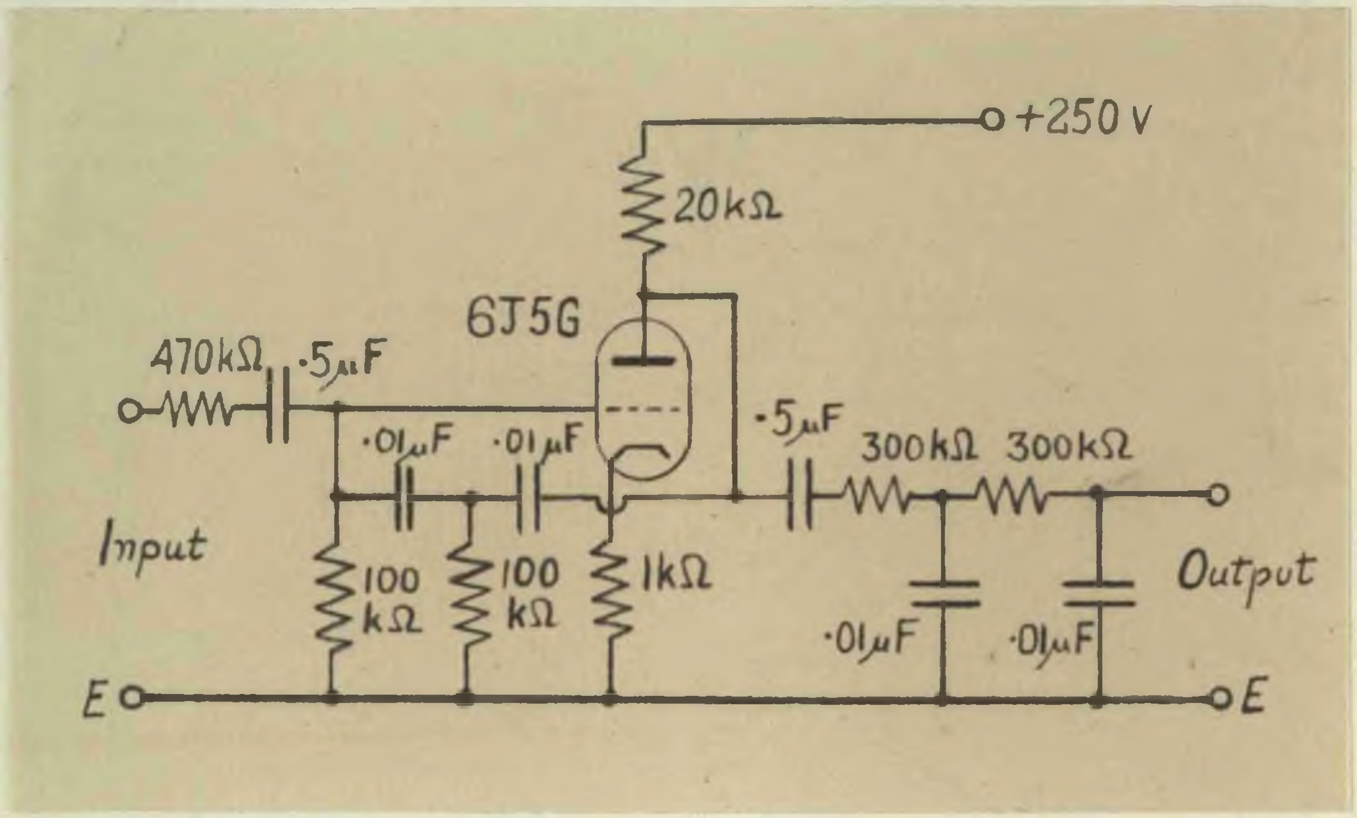

Fig. 37. Electronic low-pass filter, f_c = 60 c/s.

9. FILTERS

When a structure contains vibrations of different frequencies it is often necessary to employ some form of filter to carry out measurements at one frequency or over a limited spectrum and to reject all other frequencies. There are four main types,- namely, low pass, high pass, band pass and band stop.

Filters for frequencies above a few hundred cycles per second are dealt with comprehensively in the literature on communications but satisfactory filters for frequencies below 100 c/s are generally quite different in design. They may be electrical or mechanical, and either or both may be necessary for particular purposes. In the electrical form, it is probably easier to use R - C rather than L - C networks, particularly for the very low frequencies.

The mechanical type is particularly valuable when the frequency of interest is contained among other signals of much greater magnitude. The filter is interposed between the structure and the pick-up head. Hence the pick-up is not overloaded nor does distortion due to excessive input occur in the matching unit between the pick-up and the rest of the vibration meter. However, such filters may be difficult to design and construct in reasonable size and weight and may be subject to variations in characteristics with temperature and other operating conditions.

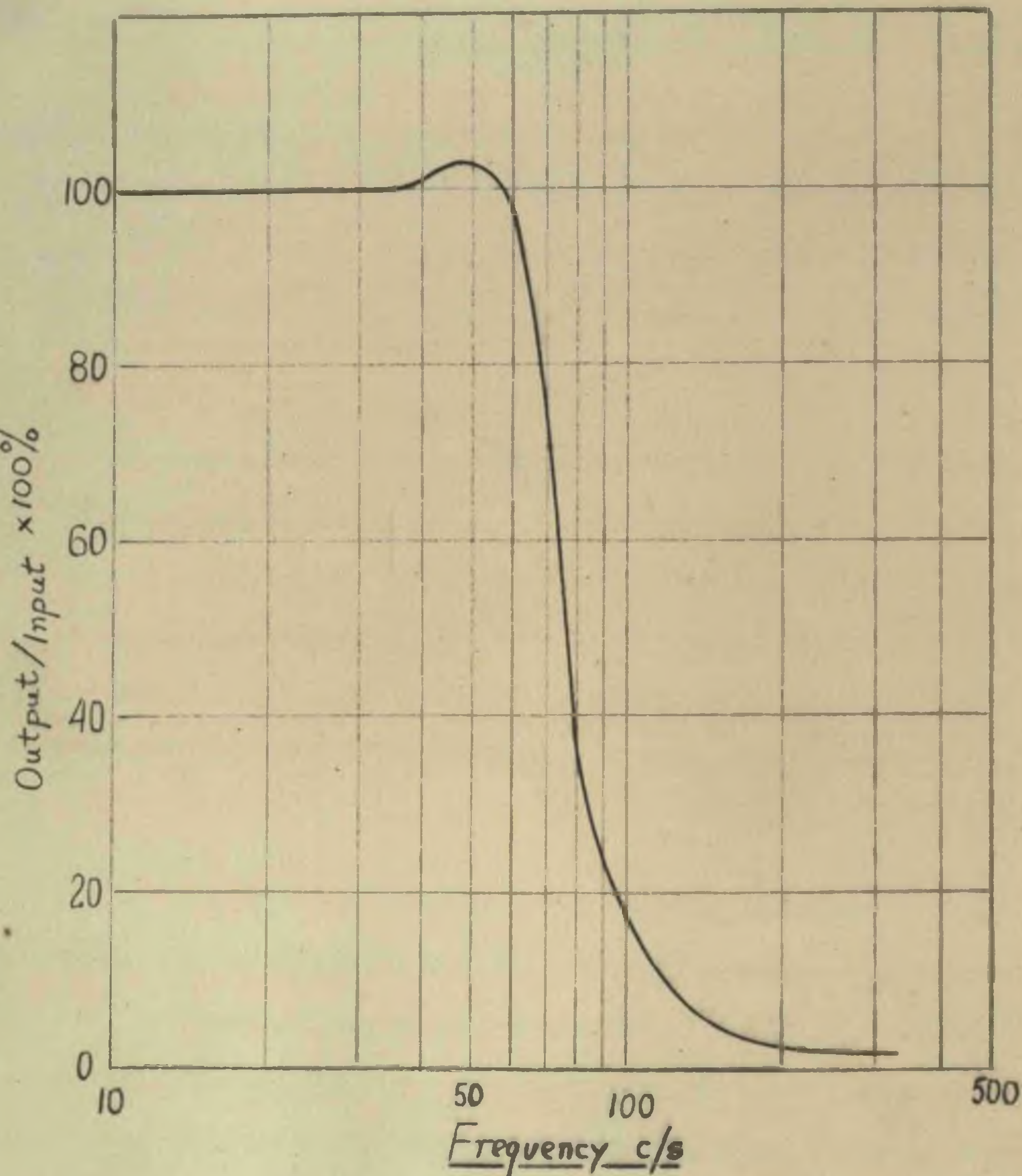

Fig. 38. Response curve for electronic low-pass filter of Fig. 37.

The electrical filter on the other hand, can be light, compact and reasonably versatile in its application.

In the course of this programme, it became apparent that if accelerometer type pick-ups were to be used for the low frequencies on board ship, then the output due to high-frequency vibrations of small displacement would have to be suppressed. For this reason, some thought was given to a practical design of filter. After some experimental work the circuit of Fig. 37 was evolved. For a wide range of cut-off frequencies an experimental unit with switched capacitors and ganged resistors is proposed. The response curve for this unit is shown in Fig. 38. It will be seen that the response is adequate for many purposes and compares very favourably with that obtainable from L - C circuits at low frequencies.

Interchanging the resistance and capacitance elements in the filter would give a high-pass unit. By connecting two tuneable filters, one high pass, the other low pass, in series or parallel, an extremely versatile band-pass or band-stop filter respectively would result. Such a device would be very useful and this circuit should lend itself to the production of a relatively cheap unit.

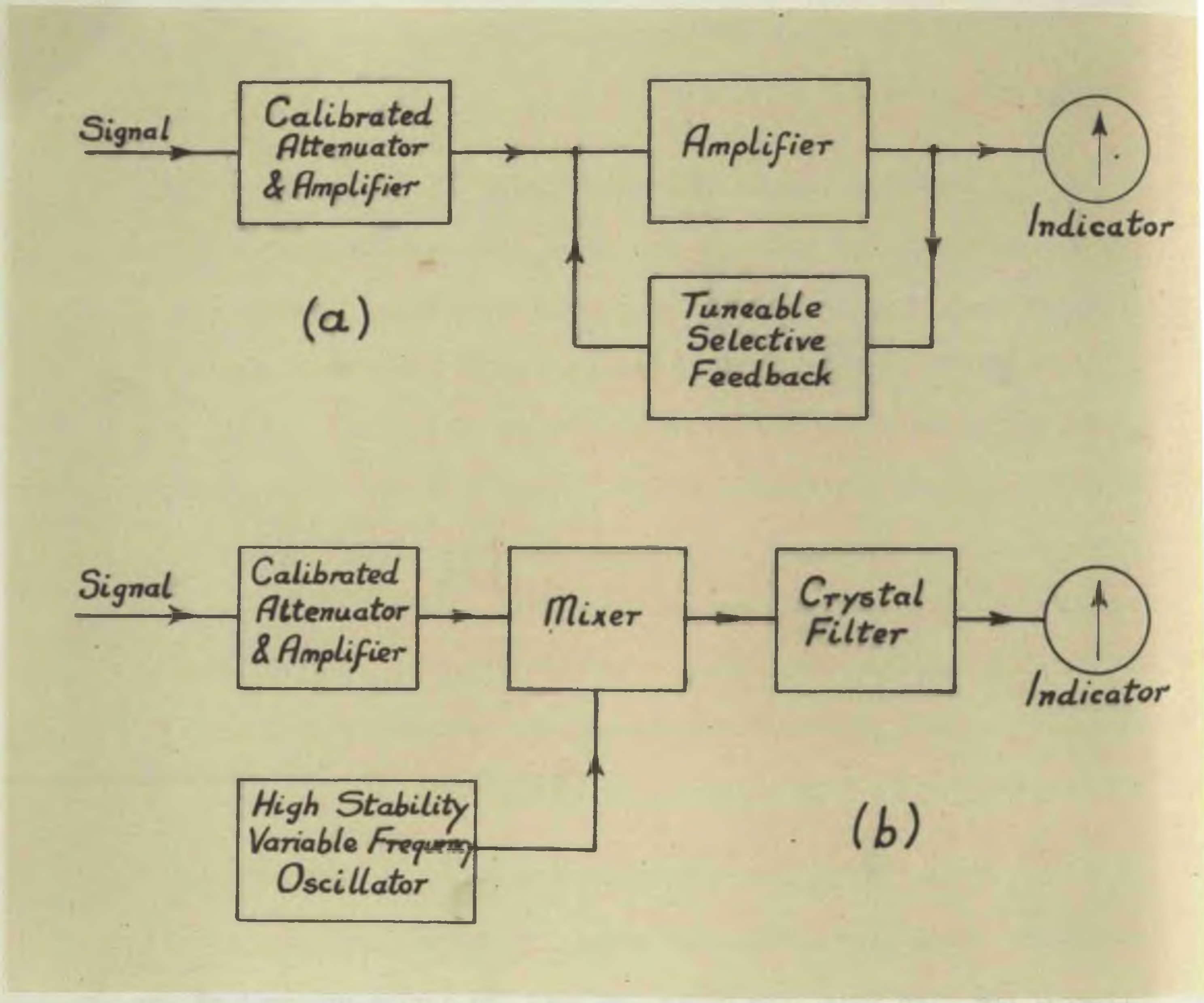

Fig. 39. Block diagram of (a) selective amplifier and (b) heterodyne forms of waveform analyser.

10. OTHER ELECTRONIC APPARATUS

The foregoing sections have dealt with equipment which was available commercially in several forms and, therefore, had to be considered critically with a view to selecting the most suitable, technically and economically. They have also dealt with the special equipment which was designed and constructed to meet the requirements of the work.

Some very useful items for a vibrations laboratory have not been discussed, and these are mentioned below.

10.1 Wave-form Analyser

Where complex waveforms are being measured this is an essential tool. In the ship-model tests, a Dawe A.F. Analyser, Type 1401C, was used, not so much for its proper purpose, but as a frequency meter. Its range is 2.5 to 7,500 c/s with an accuracy of ± 2%.

For vibration work where the frequencies of interest are usually low, the selective amplifier type of analyser, Fig. 39(a), is generally used. The heterodyne type, Fig. 39(b), has a lower frequency limit of about 30 c/s. By arranging an automatic sweep of the tuning, either mechanically or electronically, the several frequency components of the vibration may be presented panoramically on a cathode-ray tube. A little experience has been gained with a commercial panoramic analyser supplied by Industrial Electronics and it is felt that, without a great deal of experience, this instrument is more qualitative than quantitative in its applications.

10.2 Electronic Counter

Several commercial forms of this instrument are now available and much of the work dealing with damping and the effect of entrained water would have been greatly simplified by its use. To obtain maximum versatility, a double purpose instrument is desirable. For normal use at the higher frequencies it is arranged to count the number of cycles in an accurately determined interval of time, say 1 or 10 seconds. For lower frequencies, the resulting accuracy would be poor, unless a very long interval was chosen and this is usually undesirable. The unknown low-frequency signal is, therefore, used to operate an electronic "gate" so that, over the period of one cycle, the unit counts microsecond pulses from an accurate crystal. By these means, all vibration frequencies can generally be measured to better than 0.1% within one second.

11. SUSPENSION OF MODEL

Initial tests were carried out with the wax models balanced on wooden knife-edges placed at estimated nodal lines. This was quickly found to be unsatisfactory when quantitative results were required. As the model has appreciable depth, due to a vertical vibration mode, there will be a horizontal displacement at the position of a knife-edge situated several inches below the neutral axis.

Models were then suspended from a gantry by systems of multiple rubber ropes located at approximate nodal positions. Plate 2 shows a typical test rig with the Mark II exciter. The employment of this "soft" suspension was found to eliminate the need for precise positioning of the supports and the number of rubber ropes may be readily changed when the mass of the model under test is grossly altered. Tests have shown that considerable changes in the position and the spring constant of the supports have no detectable effect on the vibration frequencies or amplitudes in the model. This verifies that the supports isolate the model from the gantry and that the loss of energy from the vibrating model to the gantry is small.

11.1 Simple Analysis of Suspension Conditions

For a body with a single degree of freedom, supported on elastic material having a linear force/displacement characteristic and no damping, the equation for the natural frequency of vibration is

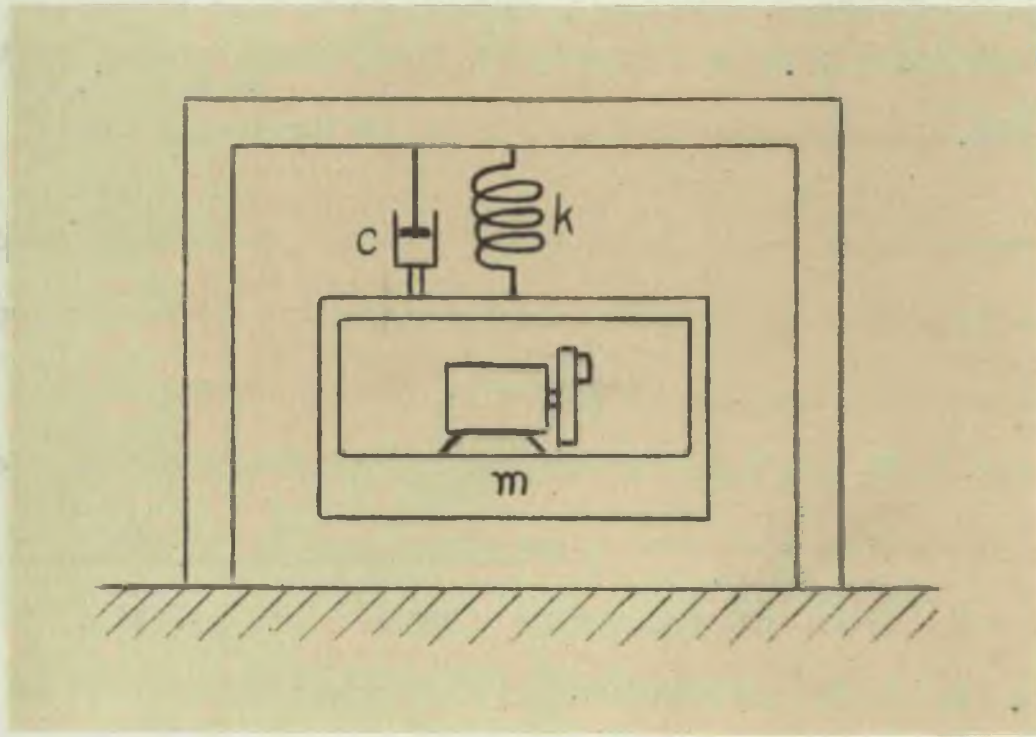

Fig. 41. Diagrammatic representation of ship model and vibration exciter suspended from gantry by damped-spring system.

$$f_o = \frac{1}{2\pi}\sqrt{\frac{kg}{W}} = 3.13\sqrt{1/\delta} \qquad (60)$$

where f_o = natural frequency (c/s)

k = spring stiffness (lb/in)

W = weight of body (lb)

g = acceleration due to gravity (in/sec^2)

δ = static deflection (W/k in)

This expression is given graphically in Fig. 40.

11.2 Theory of Isolation

Fig. 41 represents the model with exciter suspended from the gantry by a spring of stiffness k and damping c.

In practice the system is somewhat different, as the multiple suspension system can give heaving, pitching and rolling movements but, for the purpose of analysis, these may be ignored as transient and irrelevant effects.

The force acting on the mass is $P = P_o \sin \omega t$ and the motion of the mass will be of the form $x = x_o \sin (\omega t + \varphi)$. A dynamic force $F = F_o \sin (\omega t + \varphi)$ will be transmitted to the gantry by the suspension and from the equation of motion previously analysed, it can be shown that

$$\frac{F_o}{P_o} = \sqrt{\frac{1 + \left(2\frac{\omega}{\omega_o}\frac{c}{c_c}\right)^2}{\left(1 - \frac{\omega^2}{\omega_o{}^2}\right)^2 + \left(2\frac{\omega}{\omega_o}\frac{c}{c_c}\right)^2}} \qquad (61)$$

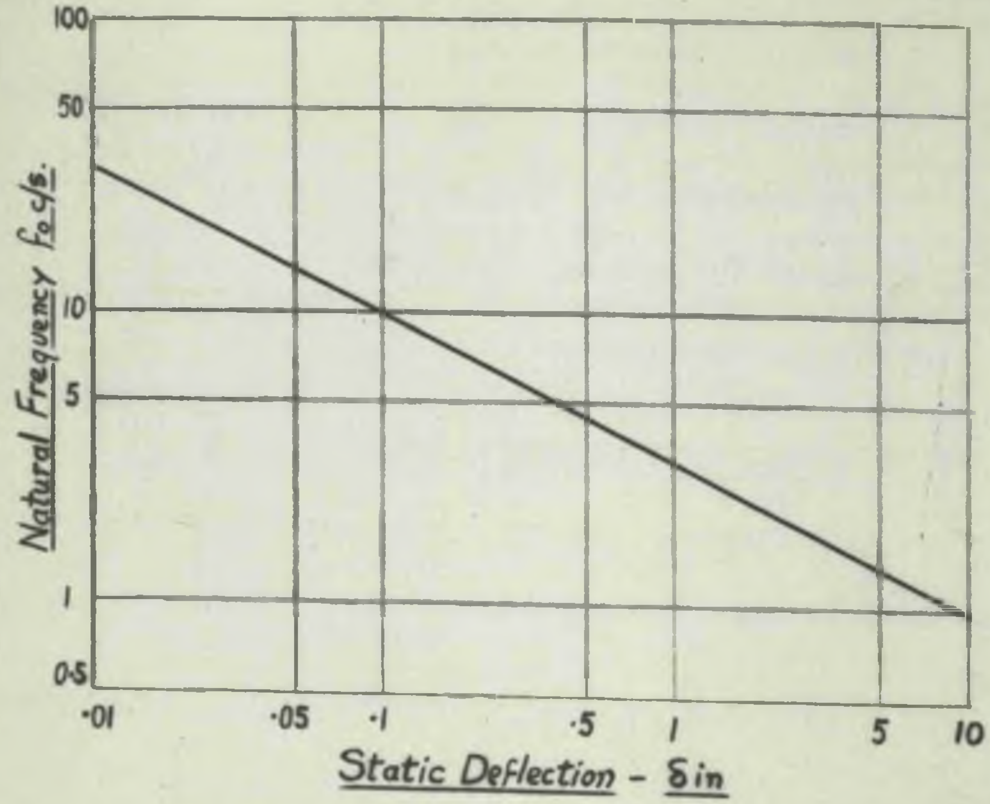

Fig. 40. Relationship between the natural frequency and static deflection of a spring-mass system.

The values of F_o/P_o or the transmission factor are plotted in Fig. 42 for various values of damping.

From these curves, it is apparent that for frequencies greater than $\sqrt{2}$ x the natural frequency of the suspension, this ratio is less than unity for all values of damping. As the value of c/c_c for the rubber is less than 0.2 and the test frequencies are at least four times the natural frequency of the suspension, less than 10% of the vibratory force at the model is transmitted to the gantry. Further, since the supports are attached at the nominal nodal lines where the displacements and forces are in practice small, the effect of these supports on resonant frequencies and damping is very small.

In practice, too, it is found that the intrinsic damping of the supports is sufficient to avoid trouble with transient effects and mode coupling. Low damping gives a more efficient support so that the optimum design for practical purposes is a compromise. No precise measurements have been made, but it would be useful to estimate the energy loss in the supports when attached to points not exactly at the nodes. Tests with varying position have indicated that the proportion of vibration energy lost to the suspension is very small. While all-rubber supports have been adequate for tests so far, it may be that with the demand for more accurate measurements, a mixture of rubber and low-loss steel spring supports may give an even better compromise.

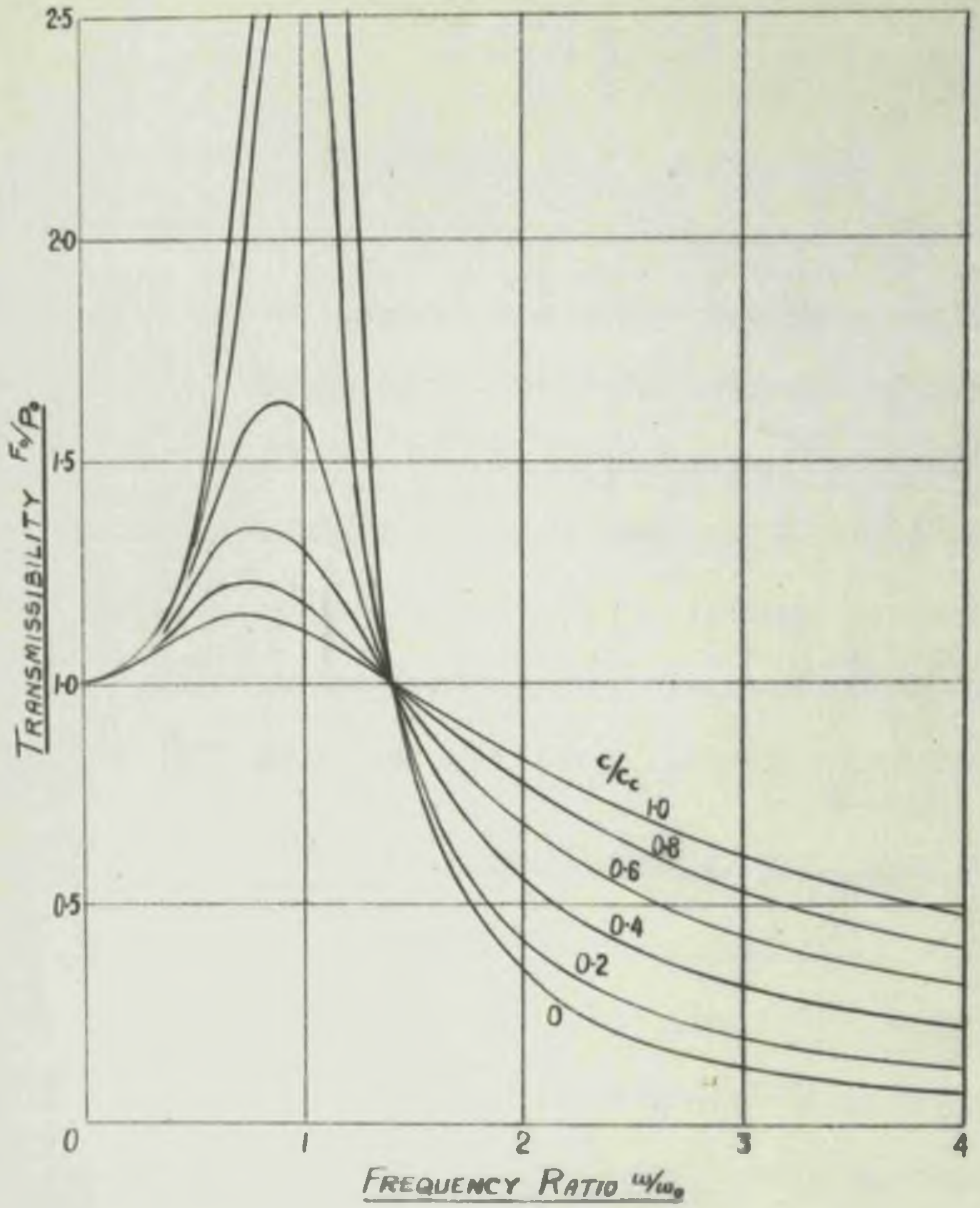

Fig. 42. Transmissibility of single-degree-of-freedom system with various values of damping.

The above discussion assumes that the displacement of the gantry is negligible. This assumption was shown to be true by applying the pick-up to various points on the gantry.

11.3 Water Tests

Tests in water were carried out with the model floating completely free, except for the flexible cables carrying the drive to the exciter and the output from the pick-up. There is a tendency for the model to drift and, if necessary, light restraint with thin rubber cords could have been applied.

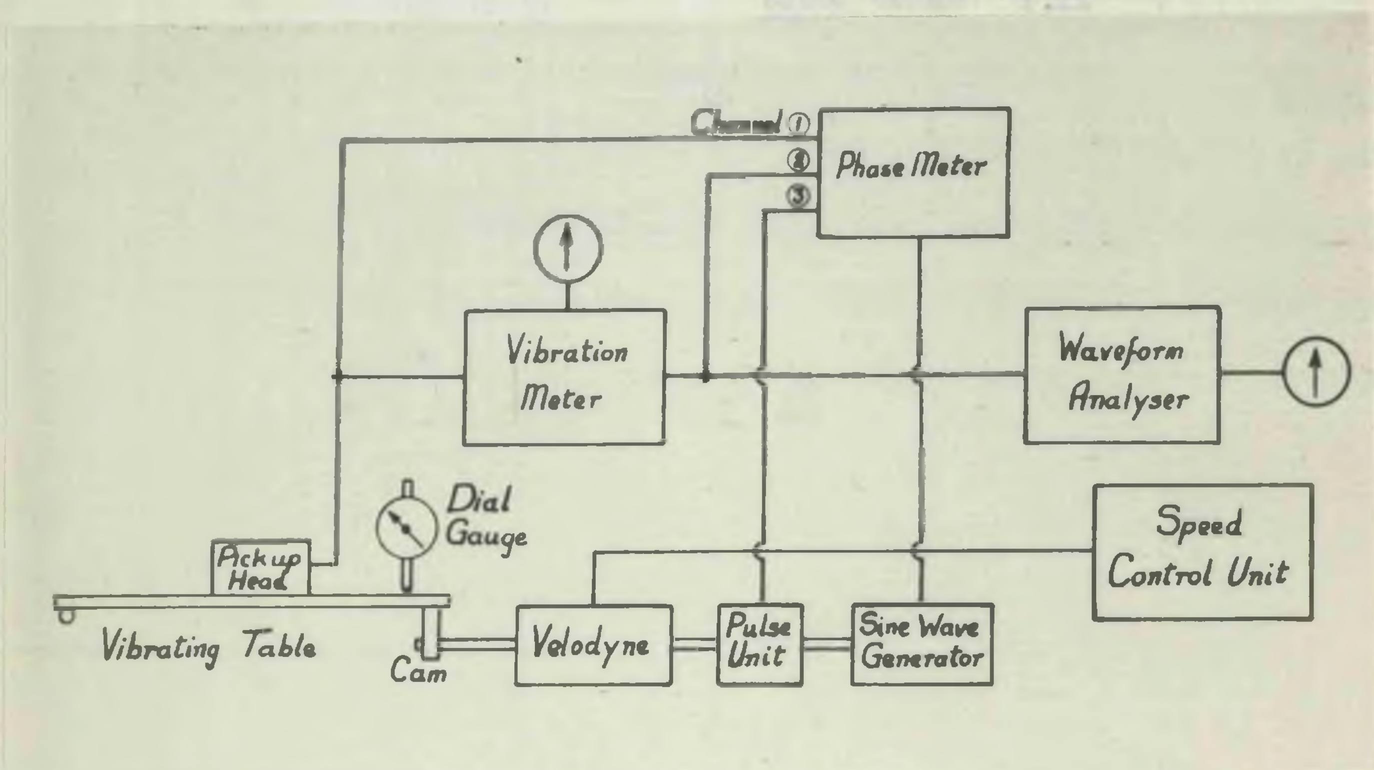

Fig. 43. Block diagram of equipment and interconnections used for instrument checking and calibration.

12. TESTING OF APPARATUS

As a number of relatively complex units were employed in the measurements, a technique for checking their calibrations and accuracy by reference to the fundamentals of length, time and mass was developed.

The equipment for this purpose is shown diagrammatically in Fig. 43.

12.1 Vibrating Table

Accelerometer type pick-ups are not amenable to static calibration and, therefore, it is necessary to calibrate them on a suitable vibrating table which can give a range of known displacements or accelerations over the required range of frequencies. It is also convenient if the table has a sinusoidal movement. Two tables for this purpose were constructed.

One of these is shown in Plate 9. A light but rigid aluminium beam is pivoted at one end and is driven at the other end by an eccentric of variable throw. In order to reduce shake and rattle to the minimum, loaded ball races are used at the driven end. The pick-up under test can be mounted at one of four pre-selected points along the beam, so that a wide range of amplitude is available. The displacement is measured by an ordinary dial gauge. This table was found to be useful for pick-ups up to 1 lb in weight.

The second table was constructed of light steel tube of square cross-section and was exceedingly rigid and free from resonances. Owing to the greater mass of this construction, it was not used at frequencies greater than about 50 c/s. By employing an eccentric of relatively large variable throw, the effect of shake at this point is minimized and small vibration amplitudes are obtained by mounting the pick-up under test near the pivoted end of the table. Shake at this end is eliminated by using crossed flexible springs as pivots.

12.2 Test Procedure

The calibration of the speed control unit was checked fundamentally by a revolution counter and stop-watch, while secondary checks were obtained from an electronic stroboscope and the waveform analyser. If an electronic tachometer had been available, this calibration could have been carried out much more accurately and without the tedium of revolution counting over relatively long periods of time.

The vibration meter was checked for displacement, velocity and acceleration readings, the amplitude and frequency at the pick-up head being known.

The phase shifts occurring in the pick-up and vibration meter can then be checked by the phase meter over the frequency range of interest. With a crystal pick-up, the input resistance (100 kΩ) to the phase meter will affect the calibration and a pre-amplifier of high input impedance and known gain and phase shift is necessary.

By the above procedure all units with the exception of the waveform analyser output level and the phase meter are calibrated. However, the construction of these two units is such that their accuracy will not normally be affected by ageing or partial failure of components.

During the period of three years that measurements were being actively carried out at Leven Shipyard, the Dawe Vibration Meter and its crystal pick-up were tested at intervals on the table and some minor adjustments were made to correct its calibration.

The wave-form analyser was checked occasionally for frequency calibration, a useful test for this purpose being to apply the output from a square-wave generator synchronised to the 50-c/s mains. This gave a quick check of amplitude and frequency at 50, 150 and 250 c/s. No trouble was experienced with this unit, but its response is such that the measurement of a second harmonic of small amplitude is not possible with any accuracy.

As the necessity for phase-measurement had not materialised during this period, no tests were carried out on the vibration meter to determine the phase-shift of the pick-up head, amplifiers and integrating units. It is thought, however, from some early oscilloscope observations that variations in these phase-shifts with frequency might have proved large and, therefore, power measurement with this equipment would have been difficult.

12.3 Other Test Methods

The simple cam- or crank-driven vibrating table is of limited use for testing at the higher frequencies and also has the disadvantage of having high-frequency harmonic noise components in its output. Even if these are small in displacement value, their acceleration is high and, therefore, can lead to very inaccurate calibration of accelerometer type pick-ups.

For these reasons, the electro-mechanical or moving-coil exciter has particular advantages, but it too must be used with care if accurate results are to be obtained. Due to the greater force/moving mass ratio obtainable in this form of exciter, it may produce up to 30 g in the frequency range 20 to 200 c/s. For higher frequencies the output is reduced and it is very difficult to measure the resulting small displacements with accuracy. Piezo-electric exciters are available for frequencies up to about 10 kc/s, but again it is difficult to obtain precise measurements of displacement.

Interferometer methods[18] have been used with success for the measurement of displacement but, even with an acceleration of 10 g at 3 kc/s, the total displacement is approximately equal to the wavelength of sodium light (23 μinch). This statement is indicative of the great difficulty of carrying out calibrations at even quite modest frequencies, particularly if low accelerations are of interest.

Even neglecting the problem of displacement measurements, there are a number of factors which must be watched if anything like an accurate calibration is to be obtained. For example, the impedance of a moving-coil exciter unit is a function of frequency and, therefore, there is often a certain difficulty in securing a match between this unit and the amplifier output over a range of frequency. As good waveform is dependent on good matching, this is a matter of importance. Resonances in the coil supports may disturb the operation and produce sideways movement.

The output from the exciter may be measured in a number of ways, all of which have certain disadvantages. An obvious and relatively simple technique is to have a signal coil which produces an output proportional to the main coil velocity. Inaccuracies in this method include variations in flux due to power supply, temperature and position changes and the effect of sideways movement. Internal mechanical resonances may also be troublesome, particularly at the higher frequencies.

If a small piece of carborundum, glass fragment or other sharp crystal-like material is attached to the moving system and brightly illuminated and viewed through a measuring microscope, it is possible, by adjusting the illuminating source, to obtain a bright line trace, whose length is the peak-to-peak amplitude of the vibration. It is not possible without elaboration to detect distortion, but sideways movement will produce an elliptical trace.

Any form of vibration pick-up could be used to calibrate the table, but accuracy presupposes that this pick-up can also be independently calibrated, statically or in some other way.

It is interesting to note that a recent American article[19] stated "at the present time, calibration accuracies of the order of 1% are limited to frequencies between 20 and 150 c/s and to accelerations below 1g. When the errors due to measurements of harmonic content, side-sway and resulting crosstalk, amplitude and frequency averaging, and relative phasing of these conditions are combined, it is not surprising to see over-all errors of ± 30% unless extreme precautions are employed in conjunction with an appropriate variety of test equipment specially engineered to meet exacting standards".

The low-frequency characteristics of the moving-coil unit are determined by the matching transformer, which becomes bulky and expensive if good waveforms are to be maintained at frequencies below 20 c/s.

13. THE PHYSICAL PROPERTIES OF PARAFFIN WAX

Before this project was commenced, some doubt was expressed whether wax ship models would vibrate and if indeed they did so, whether the intrinsic damping of the material would not be too great for useful results to be obtained. Early tests showed that this fear was unjustified and that wax behaved under dynamic conditions as an elastic material. No values for the constants were listed in the various reference books consulted, so a short investigation was carried out to determine them.

The wax as supplied is black in colour and has a density at 20°C of 57 lb/cu.ft. or 0.033 lb/cu.in.

13.1 Young's Modulus

An initial attempt was made to measure Young's Modulus of Elasticity by loading a wax bar of rectangular cross-section supported on knife edges near its ends and measuring the resulting deflection. This attempt at evaluating E was unsuccessful, as the static deflection was subject to creep even with very low stresses. Recourse was now made to dynamic methods, as these would give conditions approximating to those occurring in the model tests. By using a vibration technique the stress may be applied rapidly, so that creep effects are minimized. This technique also enables some estimation of the internal damping to be made.

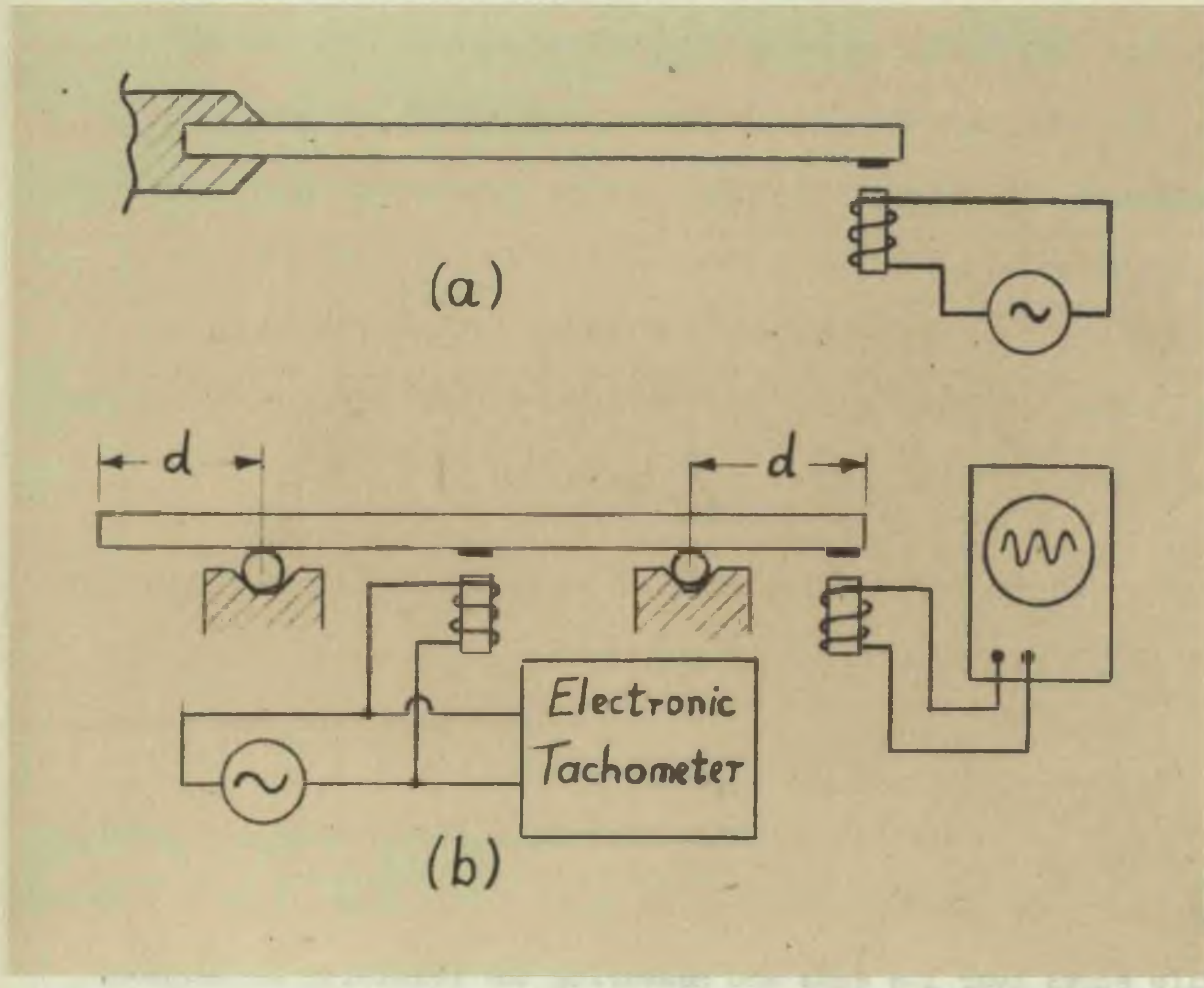

Fig. 44. (a) Cantilever and (b) two-node free-free arrangements for testing wax bars.

The problem now resolves itself into two parts, namely, a choice between longitudinal and transverse vibration modes in a bar, probably of rectangular cross-section and a choice between a number of different methods of exciting and detecting the vibrations.

Fig. 44(a) shows the arrangement first adopted. A wax bar, 24" x $1\frac{1}{2}$" x 1", was clamped in a steel framework and vibrations excited by a telephone headpiece acting on a small piece of transformer lamination melted into the bar. Vibrations at the fundamental, first and second overtones were excited and the frequencies were sharply defined and detected by touch. These tests were not considered very accurate, due to doubt about the clamp conditions and due to doubt about the frequency calibration and drift of the driving oscillator, which was of the beat-frequency type. There was some scatter in the several values of E so obtained. The mean value was about 2×10^5 lb/sq.in. which was considered to be a good first approximation.

Recent tests, using a stable oscillator with the bar vibrating in a two-node, free-free condition have given more consistent results. In these tests very small pieces of transformer lamination were used, so that the mass distribution and stiffness were not appreciably affected. The vibration was detected by a second telephone headpiece and measured on a cathode-ray tube. Frequencies were measured by an electronic counter. Support effects were minimized by resting the bar on soft rubber rods about $\frac{1}{2}$" in diameter at the approximate nodal positions. Fig. 44(b) shows the arrangement diagrammatically.

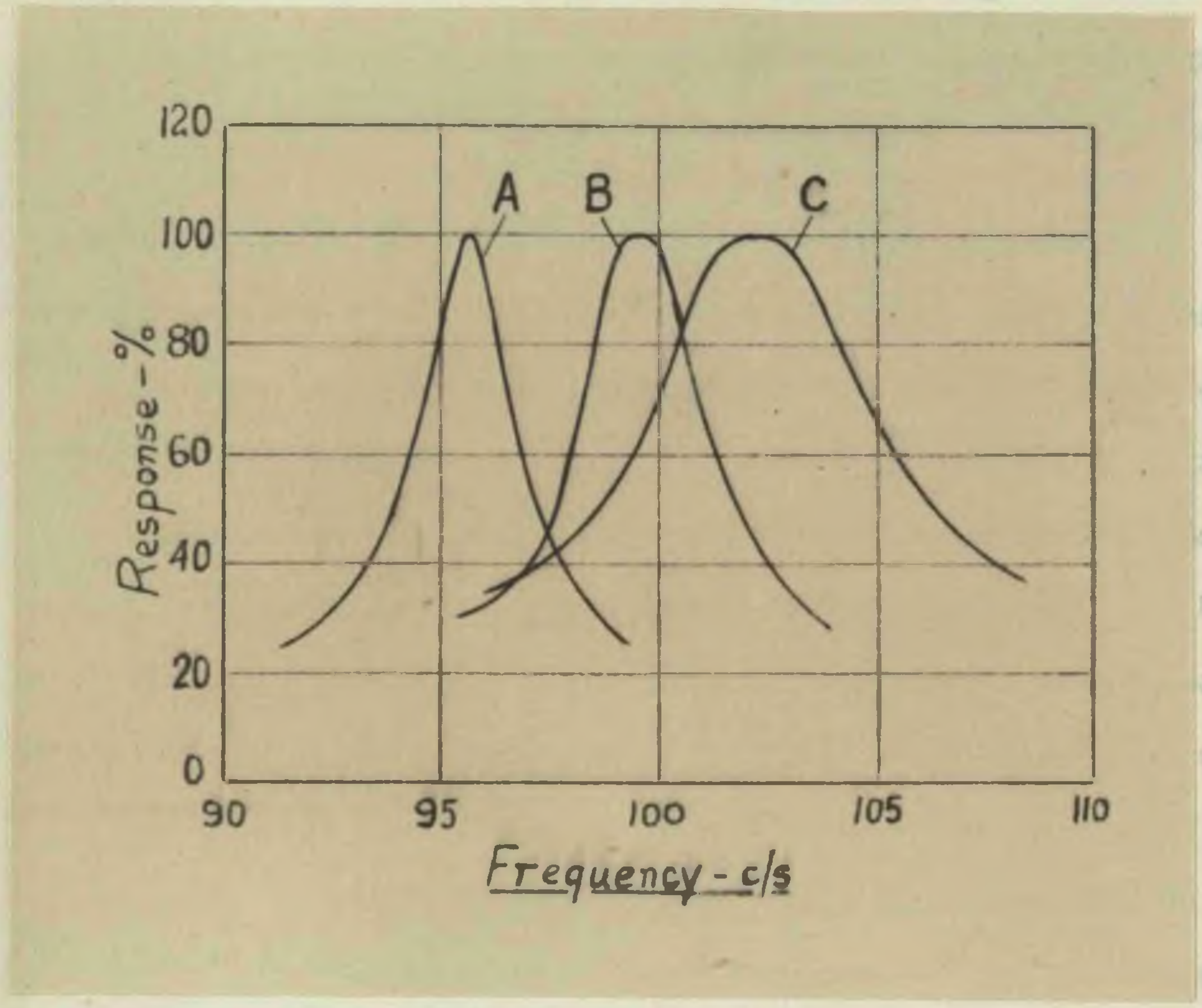

Fig. 45. Typical frequency responses of wax bar for various positions of the supports. "d" = $5\frac{1}{2}$, 6 and $6\frac{1}{2}$ inches for curves A, B and C respectively.

In order to obtain a number of values two bars 24" x $1\frac{1}{2}$" x 1" and 24" x 1" x 3/4" nominal size were tested in two positions. The mean value of E from the four values thus obtained is 2.47×10^5 lb/sq.in.

Detail figures for the tests are given in Table 2. The ambient temperature during these tests was 19°C.

Table 2

	Bar 1		Bar 2	
Breadth in	1.02	1.50	0.73	0.98
Depth in	1.50	1.02	0.98	0.73
Frequency c/s	138	95	95	74
E 10^5 lb/sq.in	2.42	2.42	2.52	2.51

Over the small range of resonant vibration amplitude which was measurable, no change in the value of E with amplitude was detected.

On the other hand, the supports had a fundamental resonance about 30 c/s and could not be considered ideal.

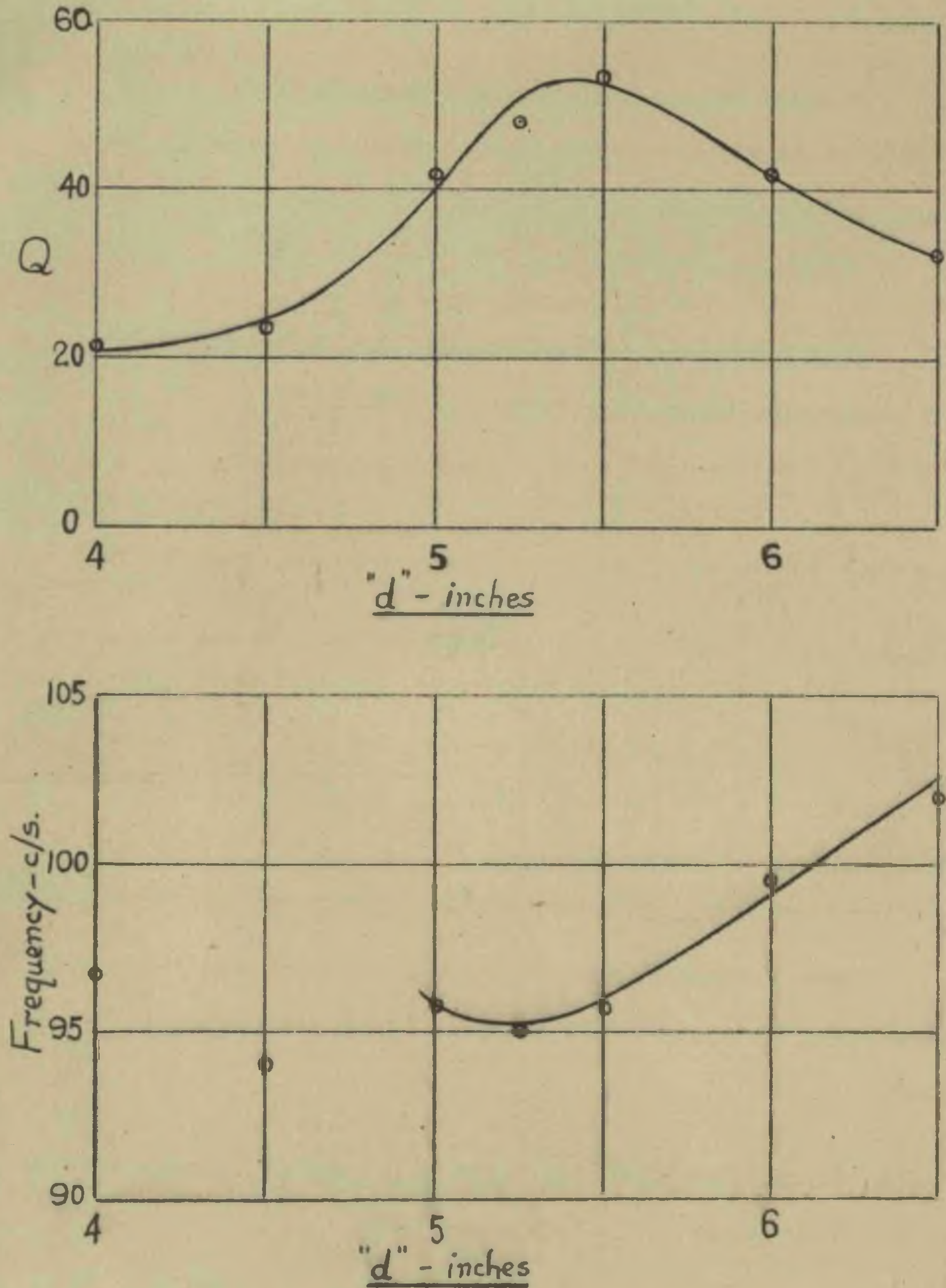

Fig. 46. Variation of Q and f_o for various positions of the supports. Theoretically nodes occur at d = .2242L = 5.39".

13.2 Damping Constant

Using the arrangement of Fig. 44(b), several resonance curves for one of the bars were plotted for different positions of the supports. These response curves are given in Fig. 45 and indicate that serious losses occur in the support system when located away from the nodes. It was also noted that temperature made a substantial difference to the damping. As facilities were not available for maintaining constant temperature, no further investigation has so far been carried out, but it is obvious that a lengthy programme of tests to determine the effect of temperature, frequency and stress on the value of E and the damping constant are necessary. It may also be that the wax is not homogeneous throughout the relative large block (up to 20 ft long) from which the ship models are machined.

From the above comments, it will be apparent that the results of these tests are suspect, but they do at least give the order of damping as $c/c_c = 0.009$ or the magnification factor Q = 55.

Fig. 46 shows the variation in resonant frequency and Q for various support positions. As these results were obtained at different times and at different temperatures, little can be said about them except that they emphasize the importance of a properly designed and constructed support system.

14. VIBRATION CRITERIA

When electrical methods of measurement are used it is simple to convert displacement, velocity or acceleration values into whichever parameter is desired by integrating or differentiating circuits. The tendency has been to express the magnitude of a vibration in terms of its displacement and this gives an immediate, if mistaken, idea of the quality of the condition, as the forces involved are proportional to the displacement and the square of the frequency. The late H.G. Yates has put forward a very good case for the adoption of the velocity criterion[20]. His arguments may be summarised in the general proposition, in which he states:

> "Mechanical vibrating systems, having geometrical similarity and constructed of the same materials, when vibrating freely in the same mode with equal linear velocities, will suffer the same vibrational stresses".

Tests on similar machines of widely differing sizes have shown that the velocity criterion holds true for relative smoothness of running and the following table gives approximate conditions.

TABLE 3

Description	Velocity in/sec r.m.s.
"Spins like a top"	0.05
Very good	0.1
Normal	0.2
Slightly rough	0.5
Unsatisfactory	1.0
Dangerous	2.0

Van Santen[21] on the other hand, appears to favour the displacement criterion and gives a similar table to which has been added a column of velocities, so that his figures may be compared with those of Yates. The figures contained in Table 4 refer only to 3000-rev/min turbo-alternators.

TABLE 4.

Description	Displacement in x 10^{-3} r.m.s.	Velocity in/sec r.m.s.
Very smooth	0.14 - 0.25	0.05 - 0.08
Good	0.29 - 0.42	0.09 - 0.13
Fair	0.45 - 0.70	0.14 - 0.21
Slightly Rough	0.73 - 1.23	0.22 - 0.38
Rough, needs correction	1.25 - 2.26	0.39 - 0.71
Very rough, immediate correction required.	Over 2.26	Over 0.71

Tables 3 and 4 show close agreement and velocity may well be adopted as the most useful criterion in specifying vibration limits. However, for laboratory tests and where structures of similar size and construction are being compared, there is no marked advantage in this criterion and displacement still has much to recommend it.

In the literature, mean, r.m.s., peak and "double-amplitude" or peak-to-peak values are quoted. Where it is made quite clear which is being used, this additional complication need not be confusing, and where the waveform is approximately sinusoidal, it is of little importance which is used. For laboratory tests dealing with sinusoidal forces and displacements, the root-mean-square values are preferred. On the other hand, the stresses in a structure are proportional to the displacements, so that peak values of displacement

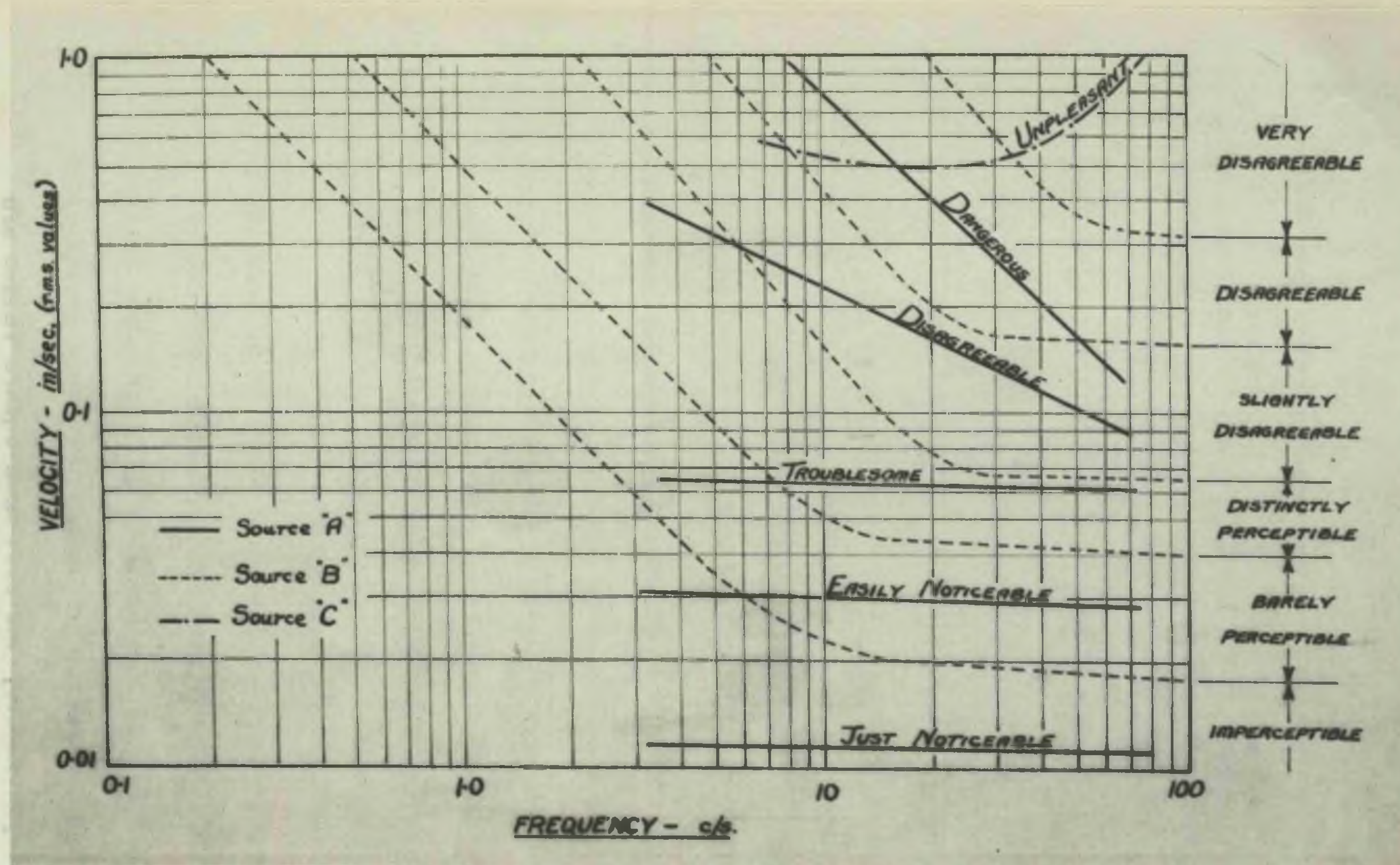

Fig. 47. Effect of vibration on humans from three sources.

provide a useful warning of excessive strains.

As mentioned in the Introduction, human comfort often influences the assessment of vibration and an attempt has been made to correlate some published and unpublished information on the effects of vibrations on humans[22,23]. The observations reported show wide divergence, possibly due to variations in the methods of test and to insufficient subjects of widely varying age, physical condition and other attributes being tested. Fig. 47 summarizes some of this information and shows that the the velocity criterion and, in fact, any other simple criterion is not widely applicable.

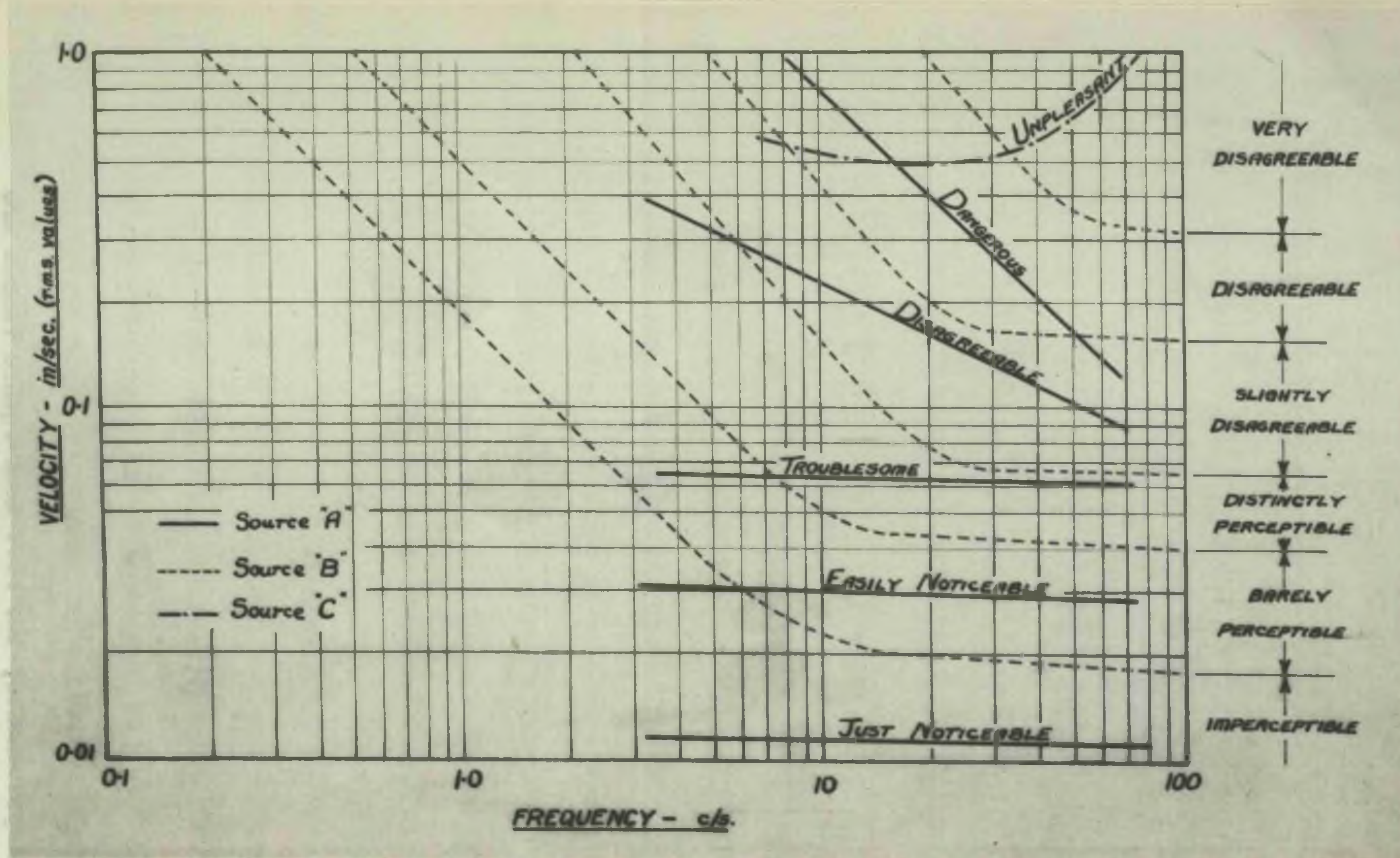

Fig. 47. Effect of vibration on humans from three sources.

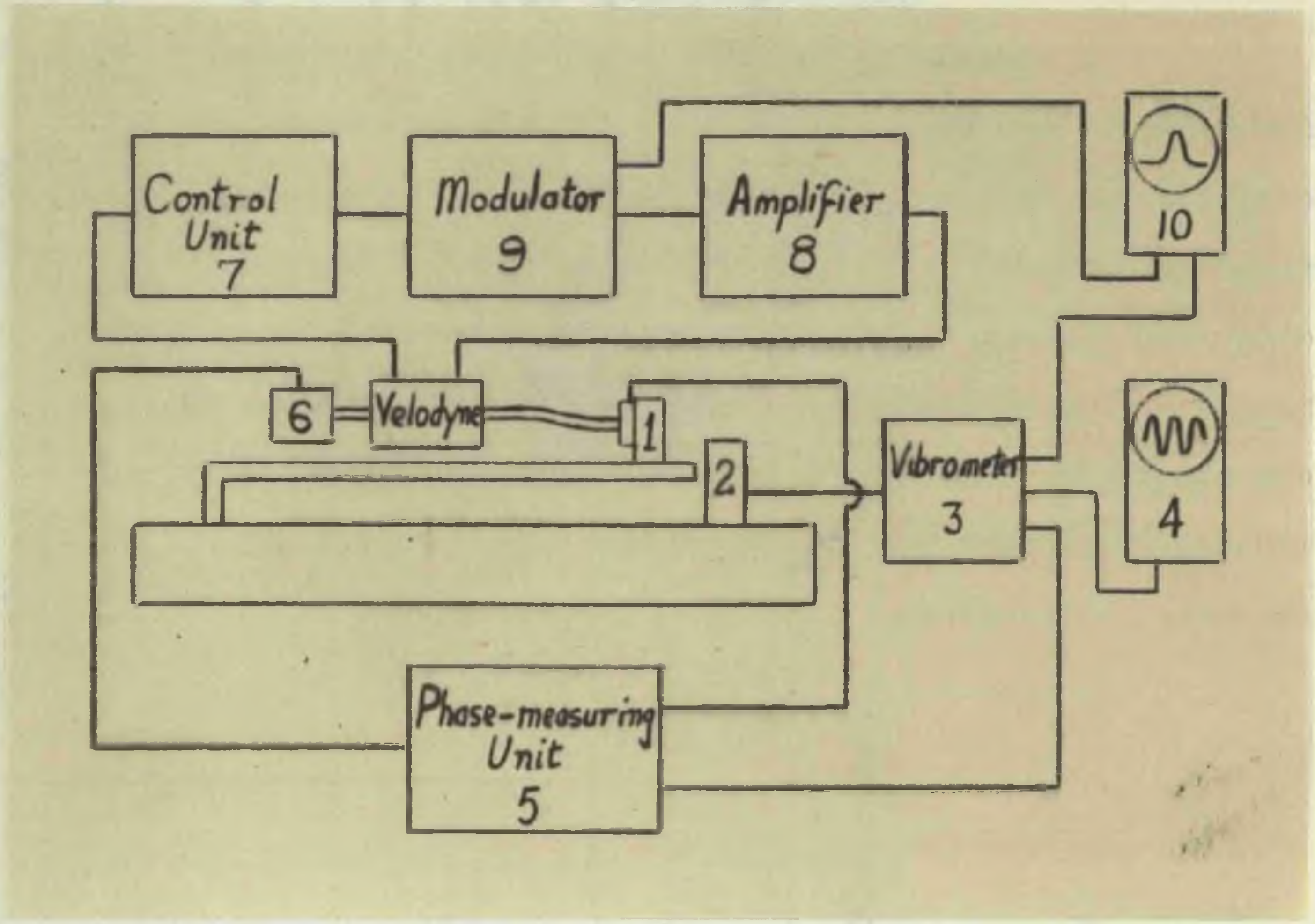

Fig. 48. Block diagram of demonstration assembly.

15. DEMONSTRATION ASSEMBLY

Plate 10 shows an assembly of electronic units for carrying out vibration tests on a steel cantilever. This equipment was demonstrated at The South of Scotland Electricity Board's Electronics and Productivity Conference and Exhibition in the Kelvin Hall, Glasgow, in February 1956.

The interconnections of the several units are shown in Fig. 48. The Mark IV out-of-balance exciter (1) is used to vibrate a steel cantilever attached to a massive bedplate at its first over-tone (about 83 c/s). The resulting vibration is detected by a balanced-transformer pick-up, (2) with its associated supply and demodulation unit, (3). Details of this unit are given in Circuit 11. A small oscilloscope (4), with a $2\frac{1}{2}$" tube shows the displacement waveform.

The phase-measuring unit (5) is fed with signals from the exciter and the vibration meter, so that it indicates the phase relationship between the exciting force and the resulting displacement. The circular time-base is produced by a two-phase alternator (6) coupled to the exciter shaft.

Interposed between the Velodyne control unit (7) and the Amplifier (8) is the Modulator (9). The construction of this unit is shown in Plate 11 and Circuit 12. A geared motor drives a two-gang, wire-wound potentiometer through a "back-lash" automatic reversing switch to produce a potential alternating linearly about zero at a frequency of about 3 cycles/minute. This potential is injected into the feed-back loop of the control system, so that the speed of the exciter is slowly scanned about its mean value by an amount indicated on the modulator instrument.

The output from the second potentiometer is applied to the X-plates of a long-persistence cathode-ray tube (10) through a d.c. amplifier. The Y-plates of this tube are fed through a second d.c. amplifier with the output from a demodulator with a long time-constant supplied from the vibration pick-up. The resulting trace on the cathode-ray tube is a plot of the vibration amplitude to a base of frequency.

This demonstration ran more or less continuously for four days and performed very well within its limitations.

16. CONCLUSIONS

When this project was commenced the only available commercial units which were considered suitable were the Dawe vibrometer and analyser. As a result, equipment was constructed to suit the job and, on the whole, it has performed adequately and reliably.

A Velodyne unit (Servomex) is now available commercially and a phase-measuring unit similar to the one constructed has been described[24]. It is significant, however, that, although a number of small pick-up heads are available and many associated vibrometer circuits are described in the literature, no manufacturer appears to market a complete unit entirely suitable for this project.

Various suggestions for future work have been made in the foregoing text and at present the greatest need seems to be for a good light-weight pick-up. This should have a total weight of less than 1 oz and, along with its associated amplifiers, be capable of covering the frequency range with reasonable sensitivity. The entire unit should be simple and stable, so that its accuracy can be relied upon without the need for too frequent calibration.

The out-of-balance exciter could be improved by reducing its size and using a light-alloy case, helical gears and ball or roller bearings capable of withstanding high loads, so that the same force outputs as at present could be obtained with less static mass.

Other equipment which would simplify some sections of the work include a precision tachometer and a multiple-channel pen recorder of adequate frequency response. The latter would be invaluable for the determination of damping by the decrement method and should greatly assist the investigation into the properties of wax. This investigation is likely to prove tedious with so many factors affecting these properties.

17. REFERENCES

Over the past few years a mass of literature has been published, dealing with instrumentation suitable for the production and measurement of mechanical vibrations. Out of all this, over 300 references have been abstracted as being of some relevance to this project. Listed below are the publications referred to in the text and a number of others which have been selected for their fundamental nature or because they contain extensive and useful bibliographies.

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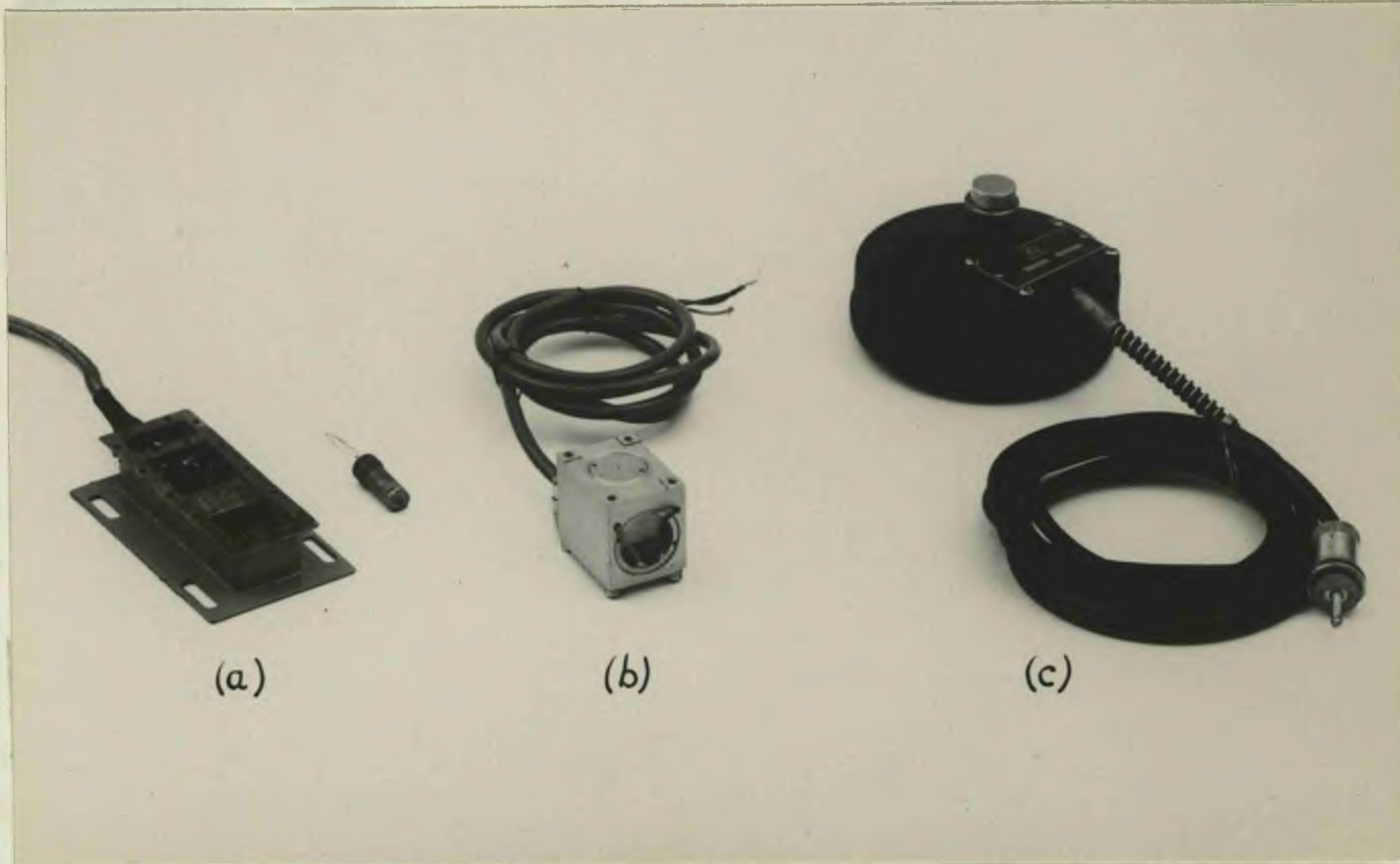


PLATE 1.

Accelerometers using (a) triode, (b) moving-core variable inductance and (c) Rochelle salt crystal transducers.

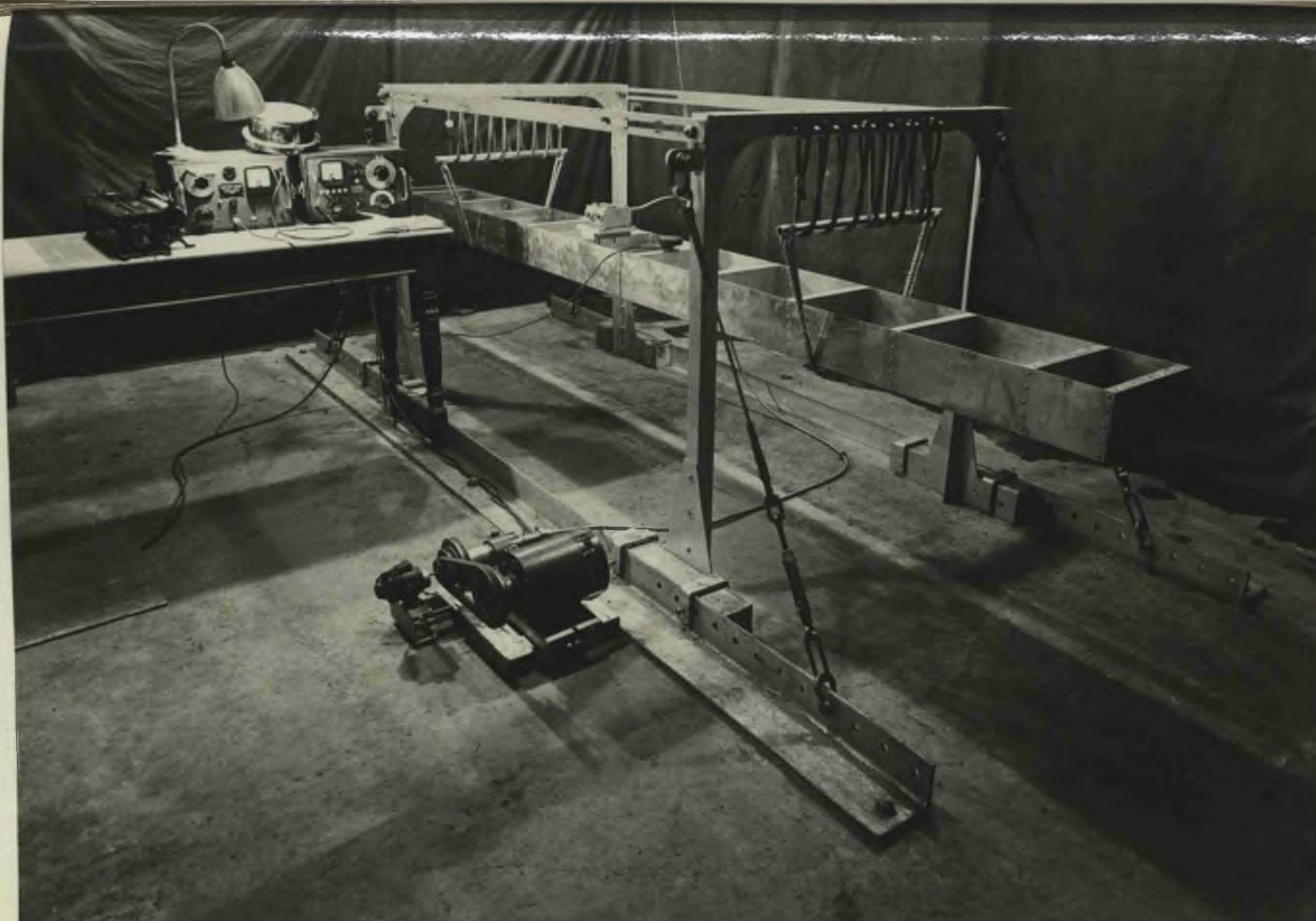


PLATE 2

Mark II Exciter with aluminium model suspended in air.

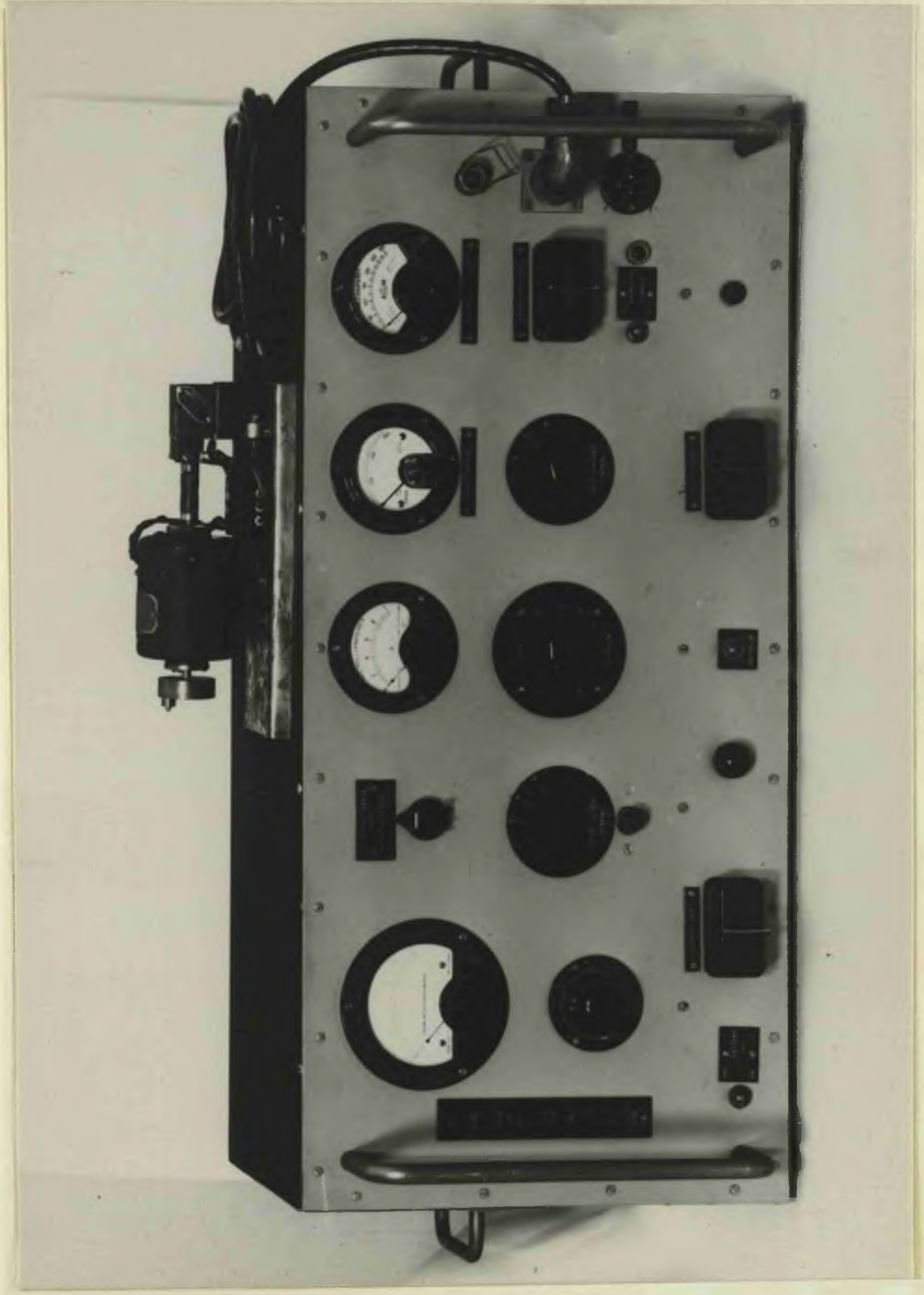


PLATE 3

Mark III Exciter and Control Unit.

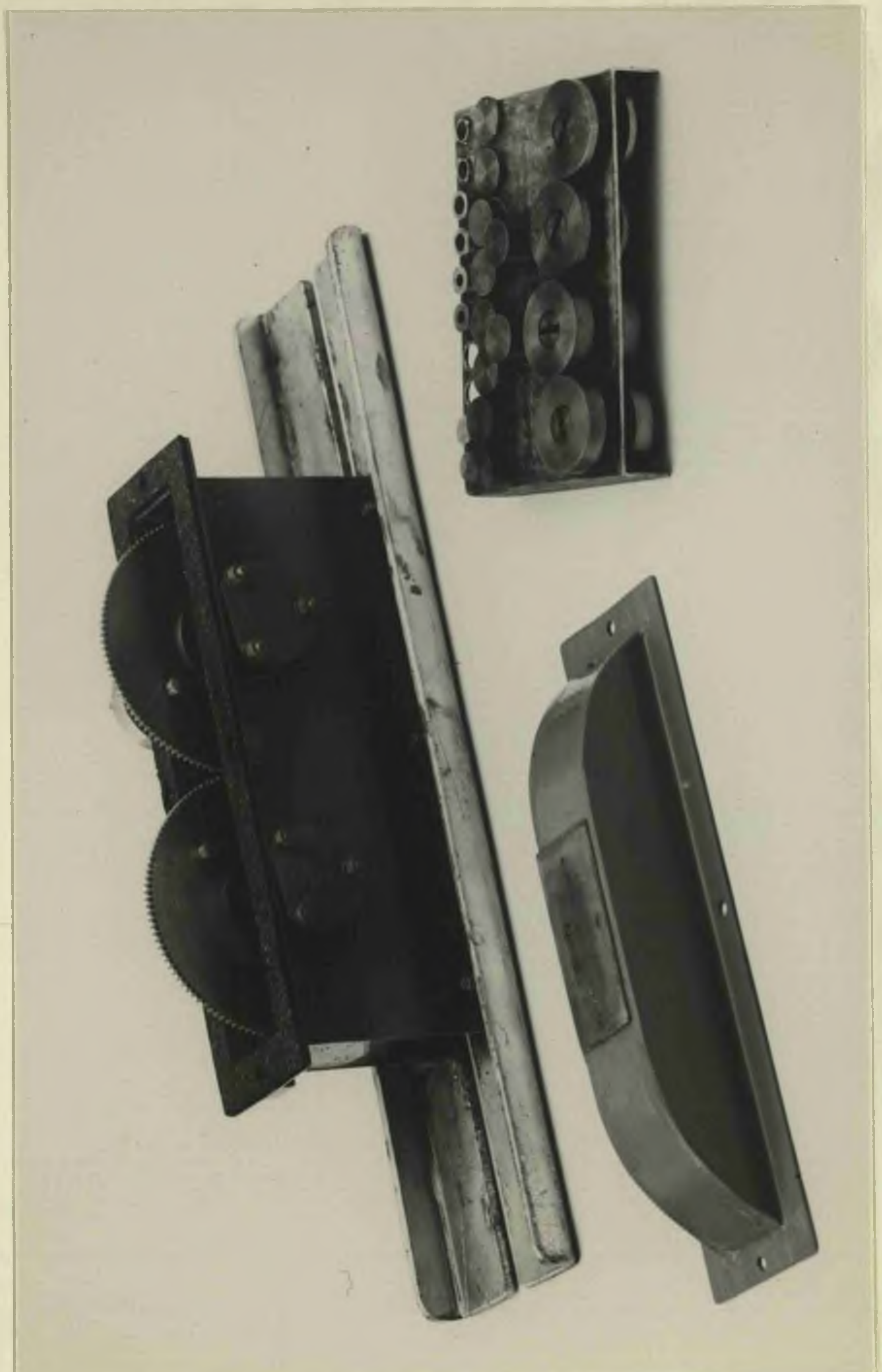


PLATE 4

Mark IV Exciter with set of out-of-balance weights.

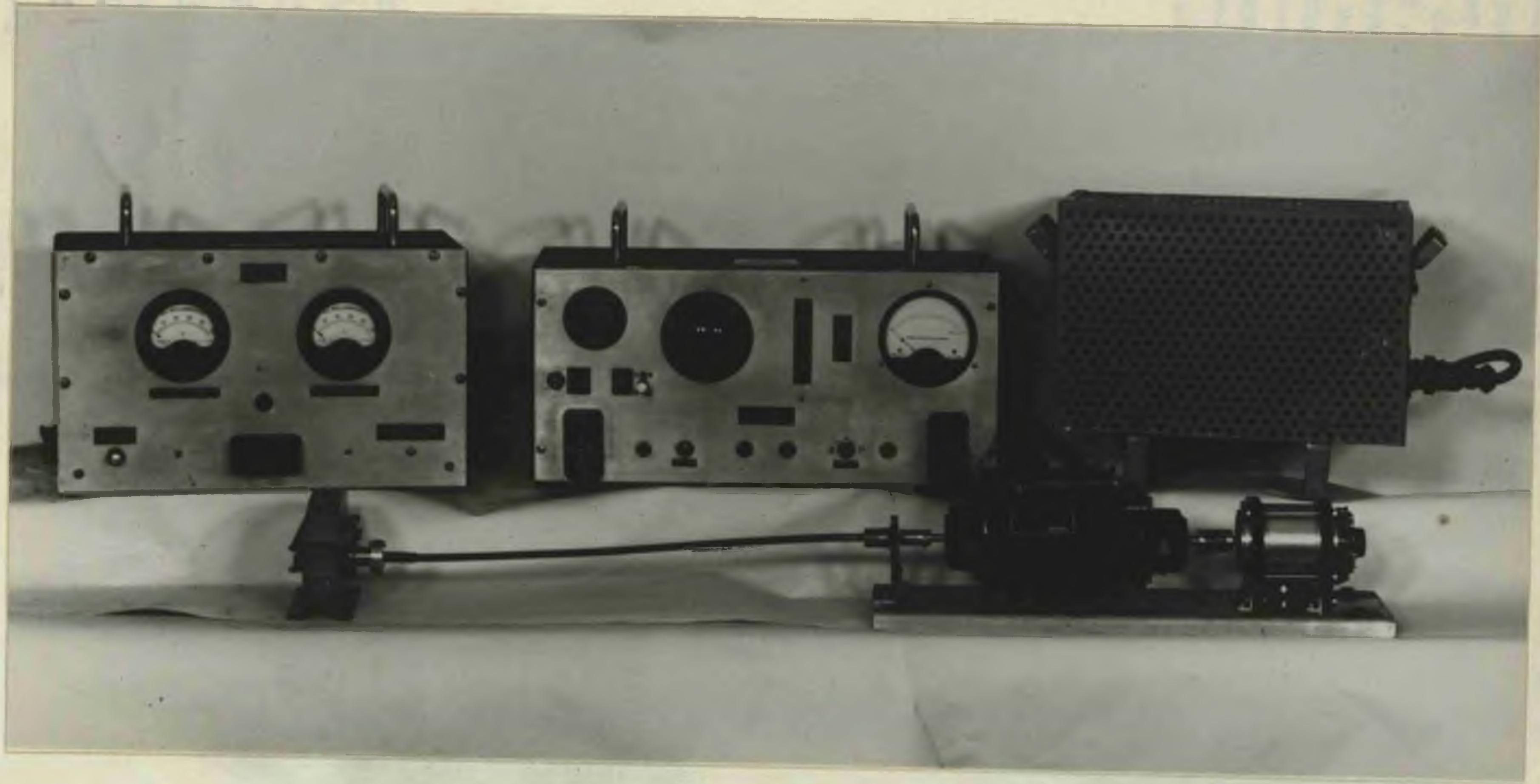
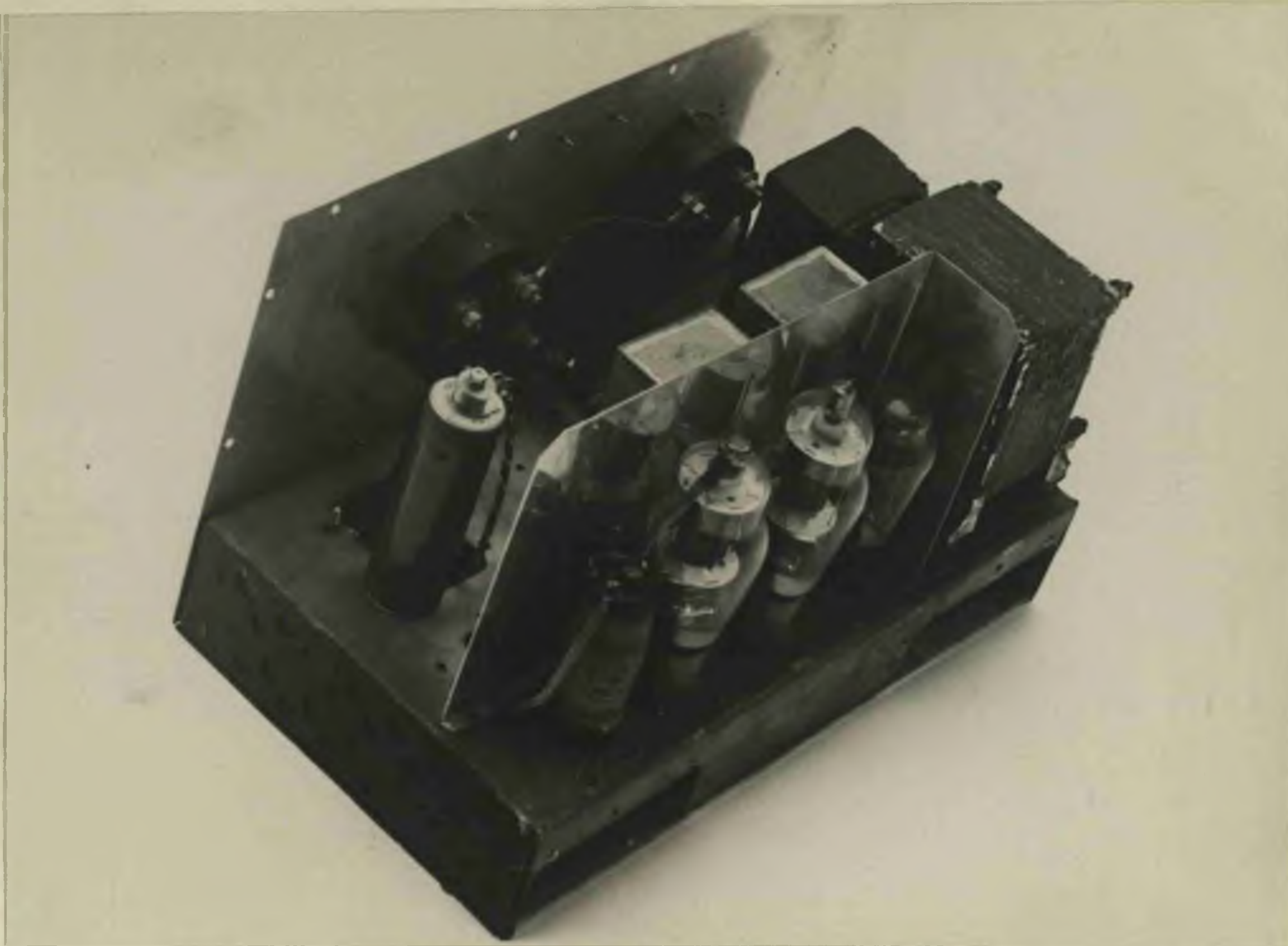


PLATE 5

Mark IV Exciter with Control, Amplifier and Resistance Units.



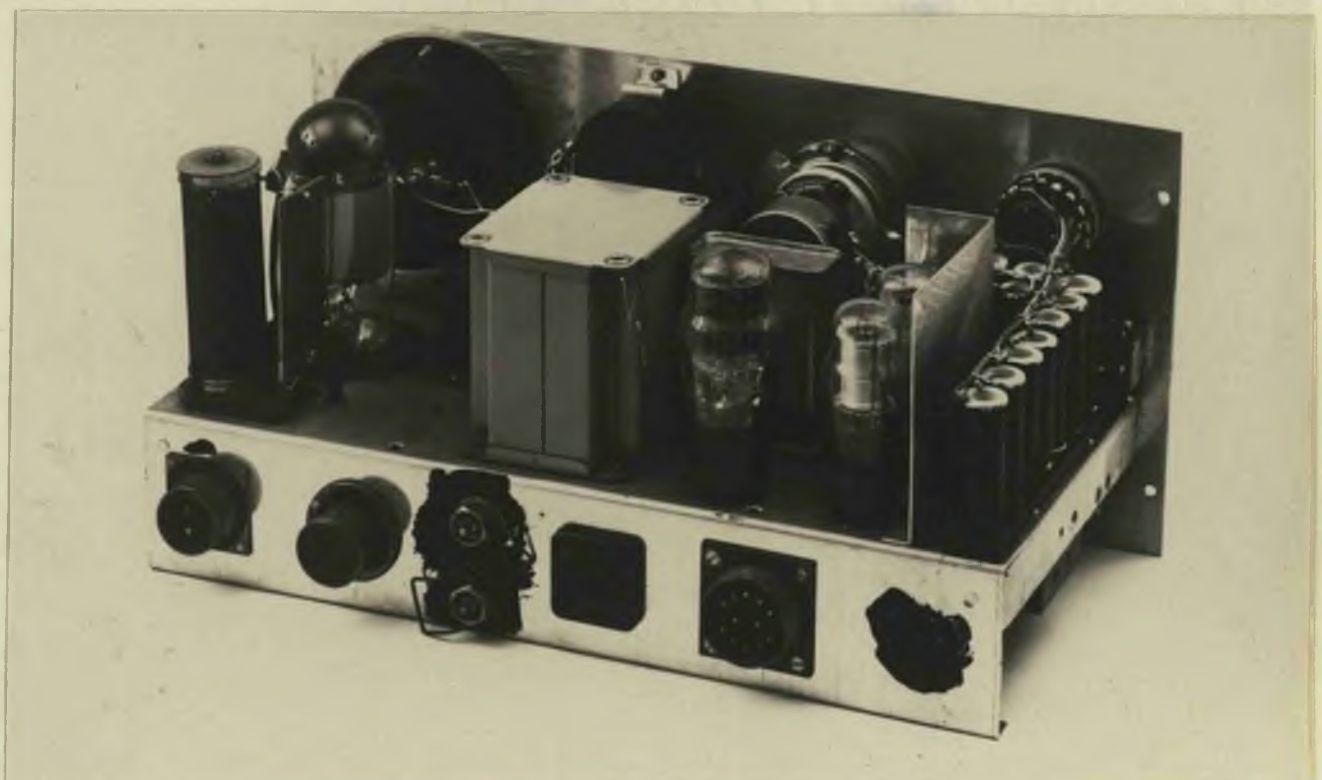
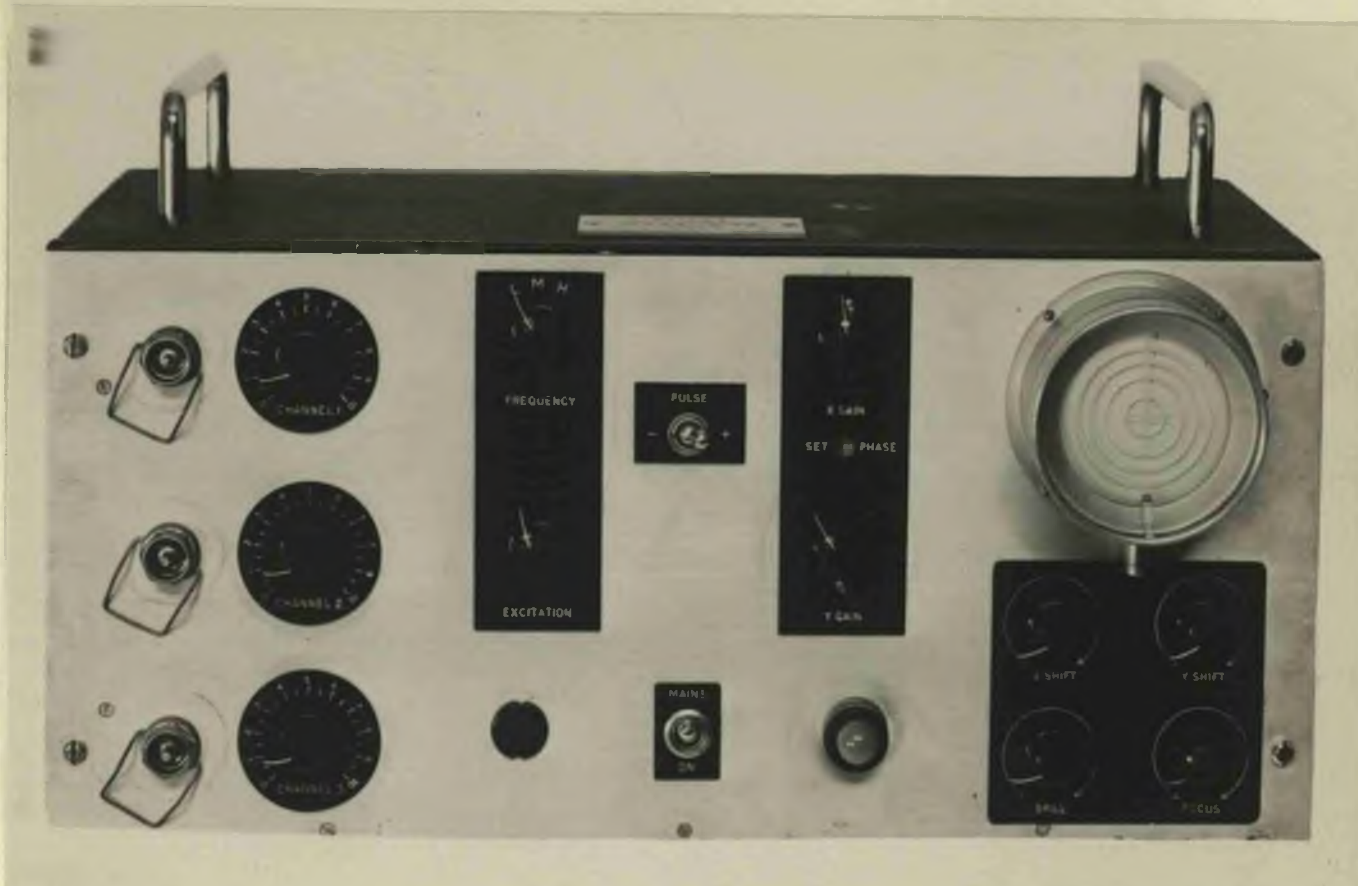


PLATE 6

(a) Amplifier for Mark IV Exciter Unit (above)

(b) Control Unit (below).



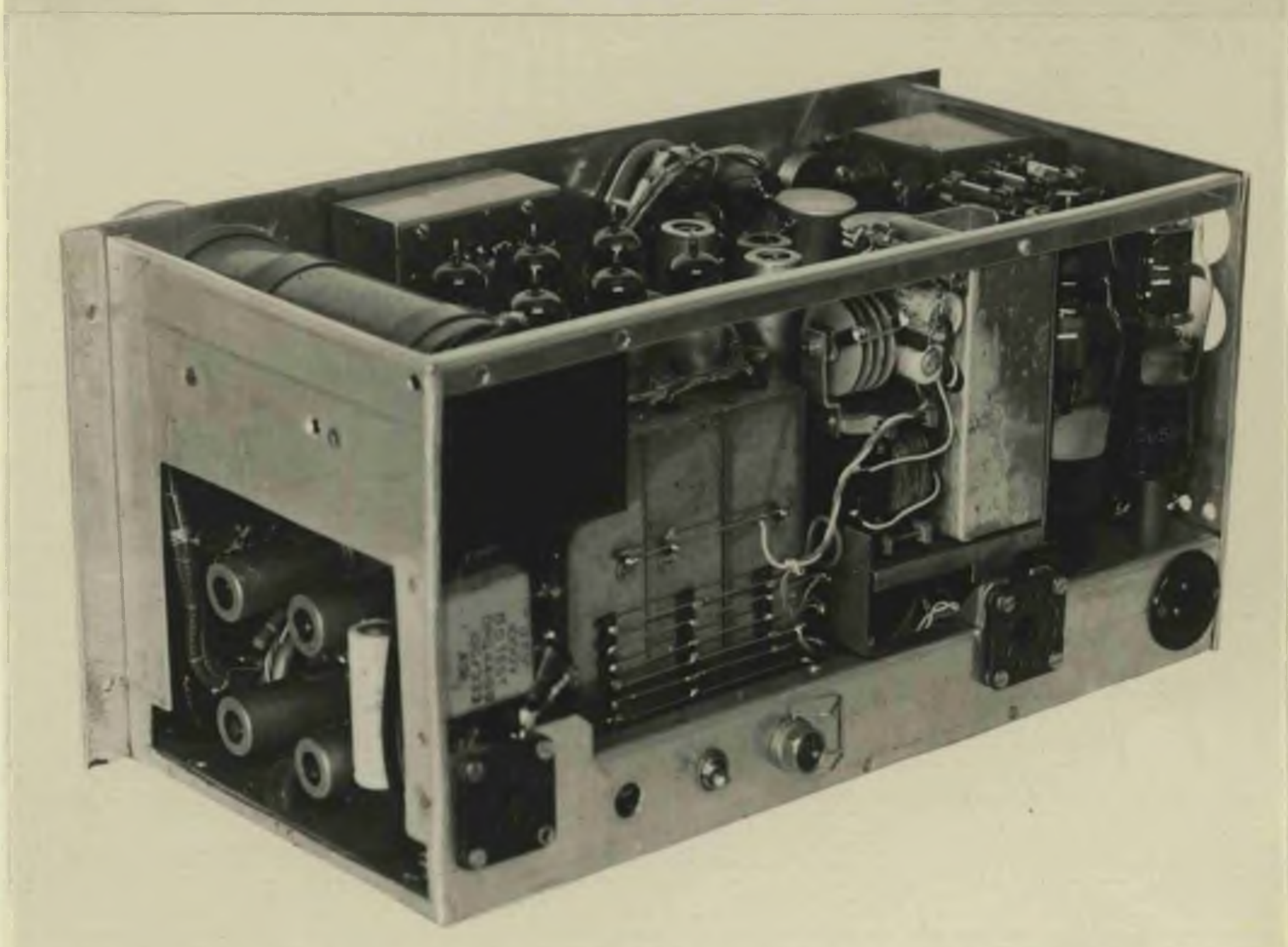
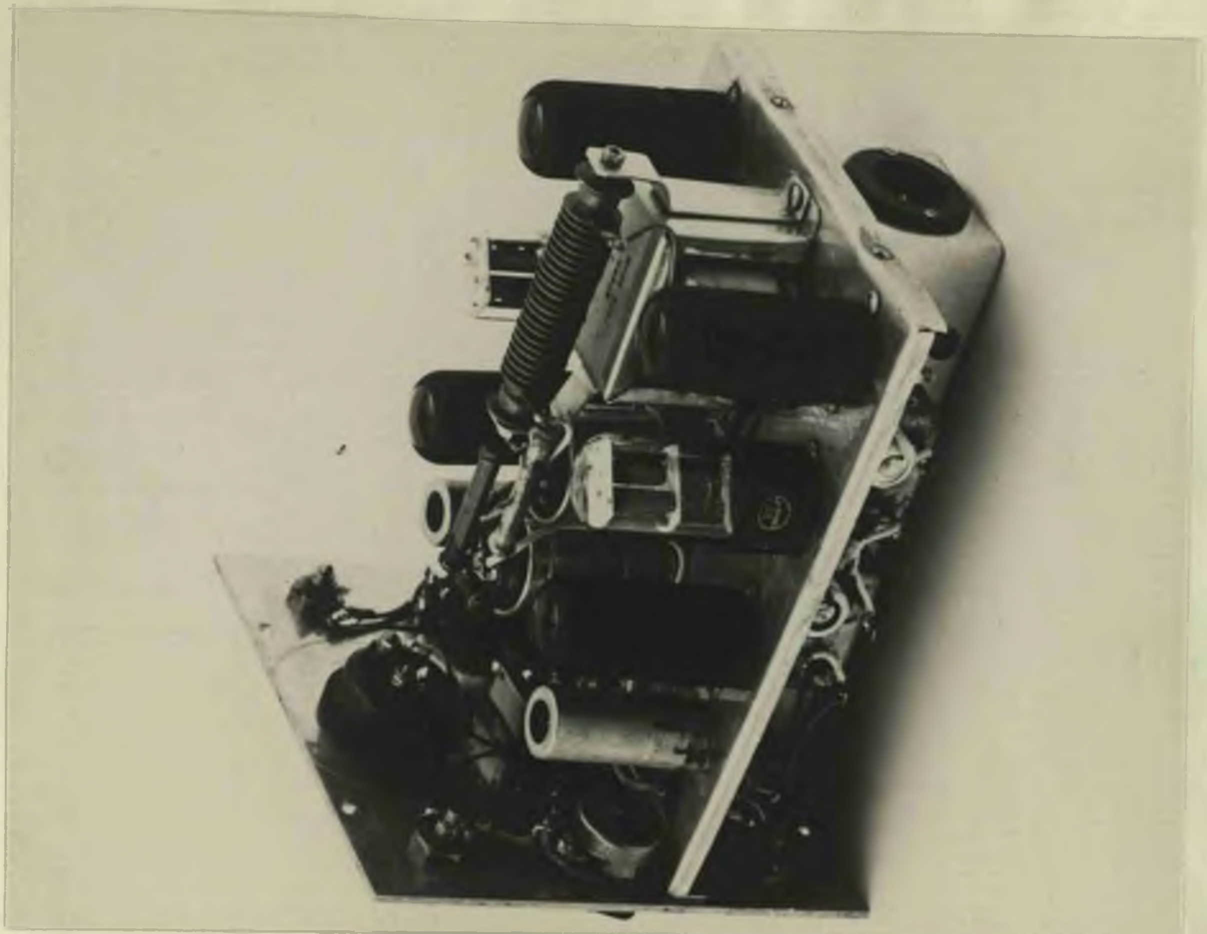


PLATE 7

Phase-measuring Unit.



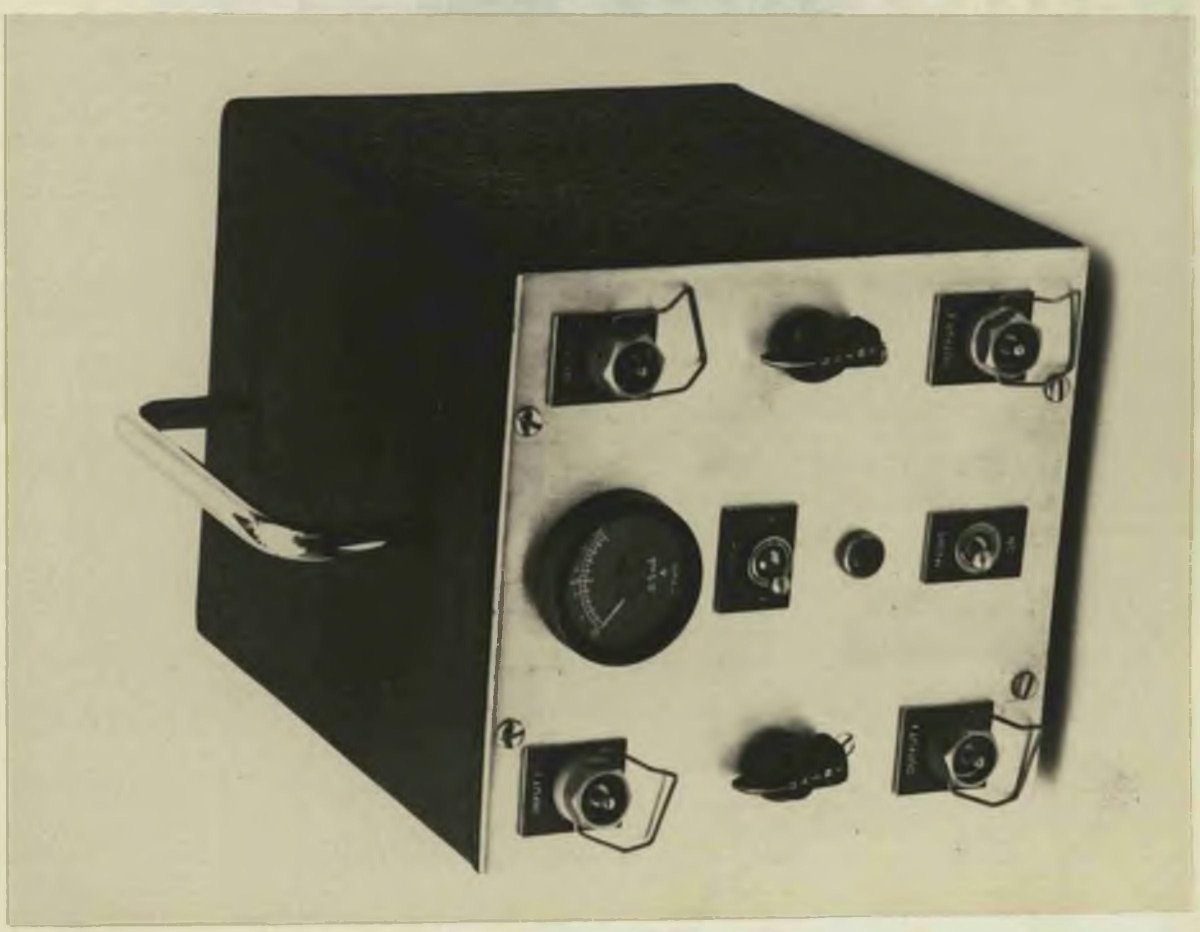


PLATE 8

Limiter.

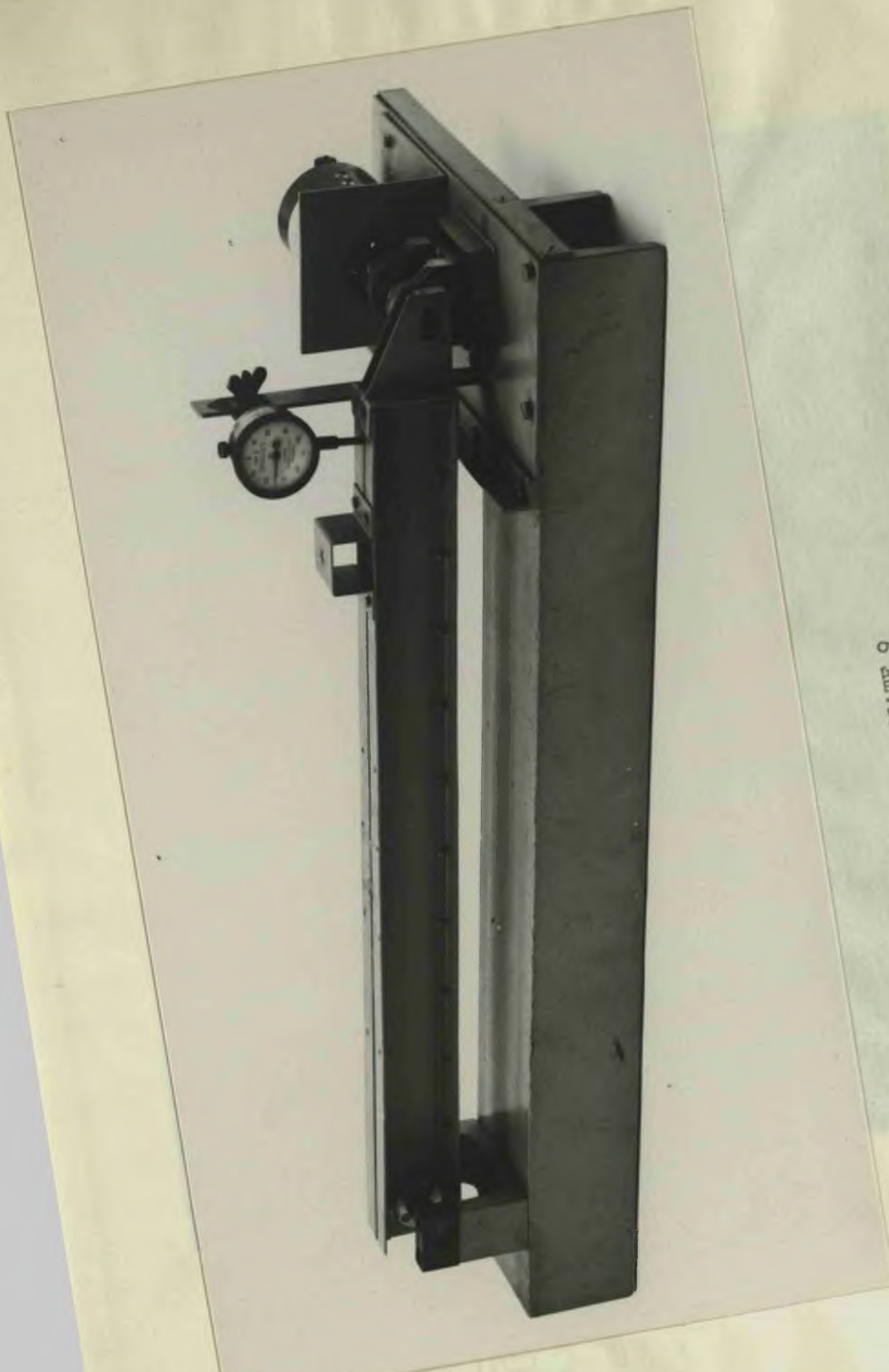


PLATE 9

Vibrating Table.

PLATE 10

Demonstration Assembly.

PLATE 11

Modulator Unit.

19. CIRCUITS

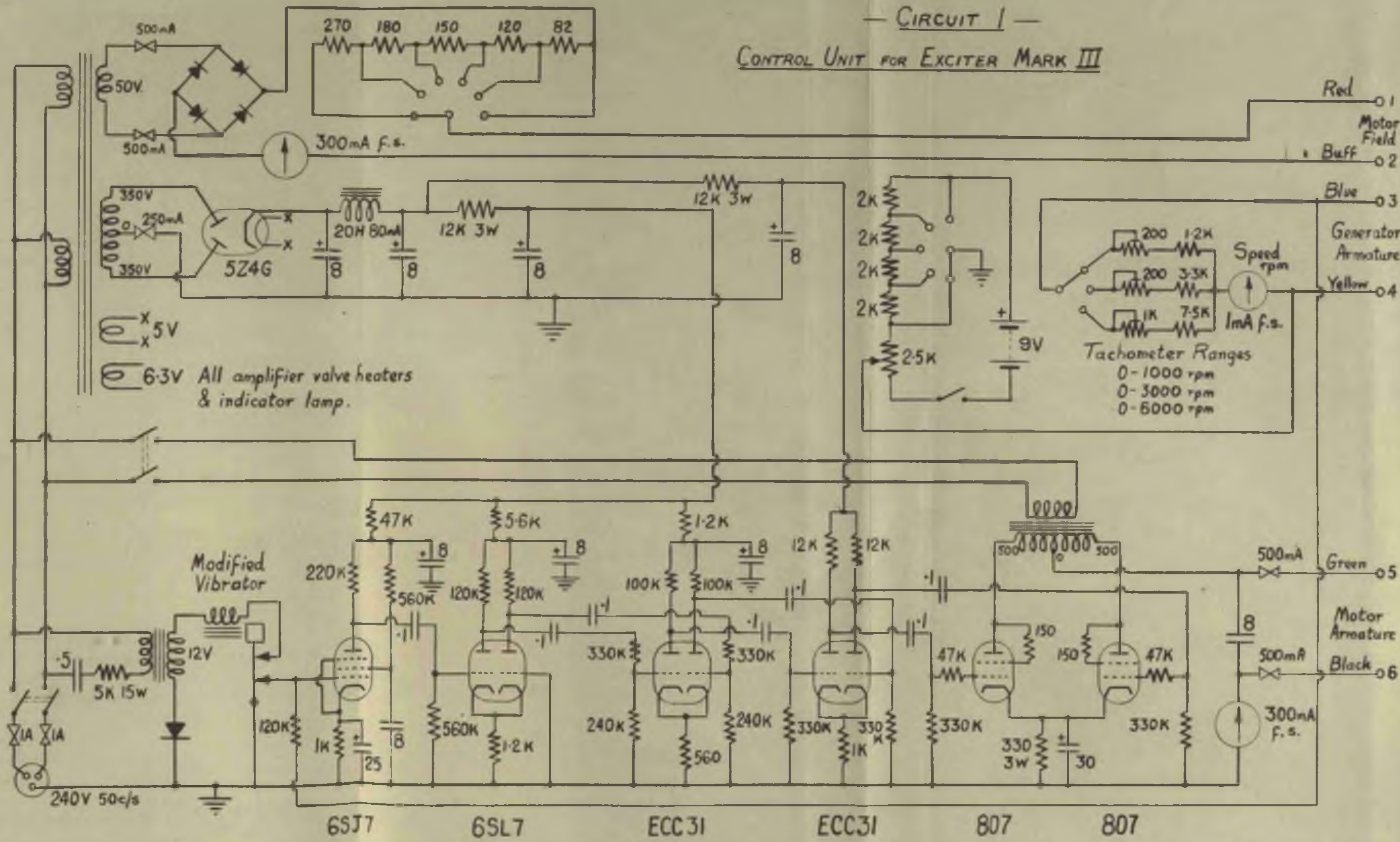
1. Control Unit for Exciter Mark III.
2. Control Unit for Mark IV Exciter System.
3. D.C. Amplifier for Mark IV Exciter System.
4. Interconnections for Mark IV Exciter System.
5. Phase-Measuring Unit. Two-phase Generator and Filter.
6. Phase-Measuring Unit. Two-phase Amplifier.
7. Phase-Measuring Unit. Signal Amplifier and Pulse-Forming Unit.
8. Phase-Measuring Unit. Pulse Amplifier.
9. Phase-Measuring Unit. Power Supplies and Cathode-Ray Tube Connections.
10. Two-Channel Limiter.
11. Balanced-Transformer Pick-up, Phase-Sensitive Demodulator and Oscilloscope Connections.
12. Modulator Unit and Timebase for Oscilloscope.

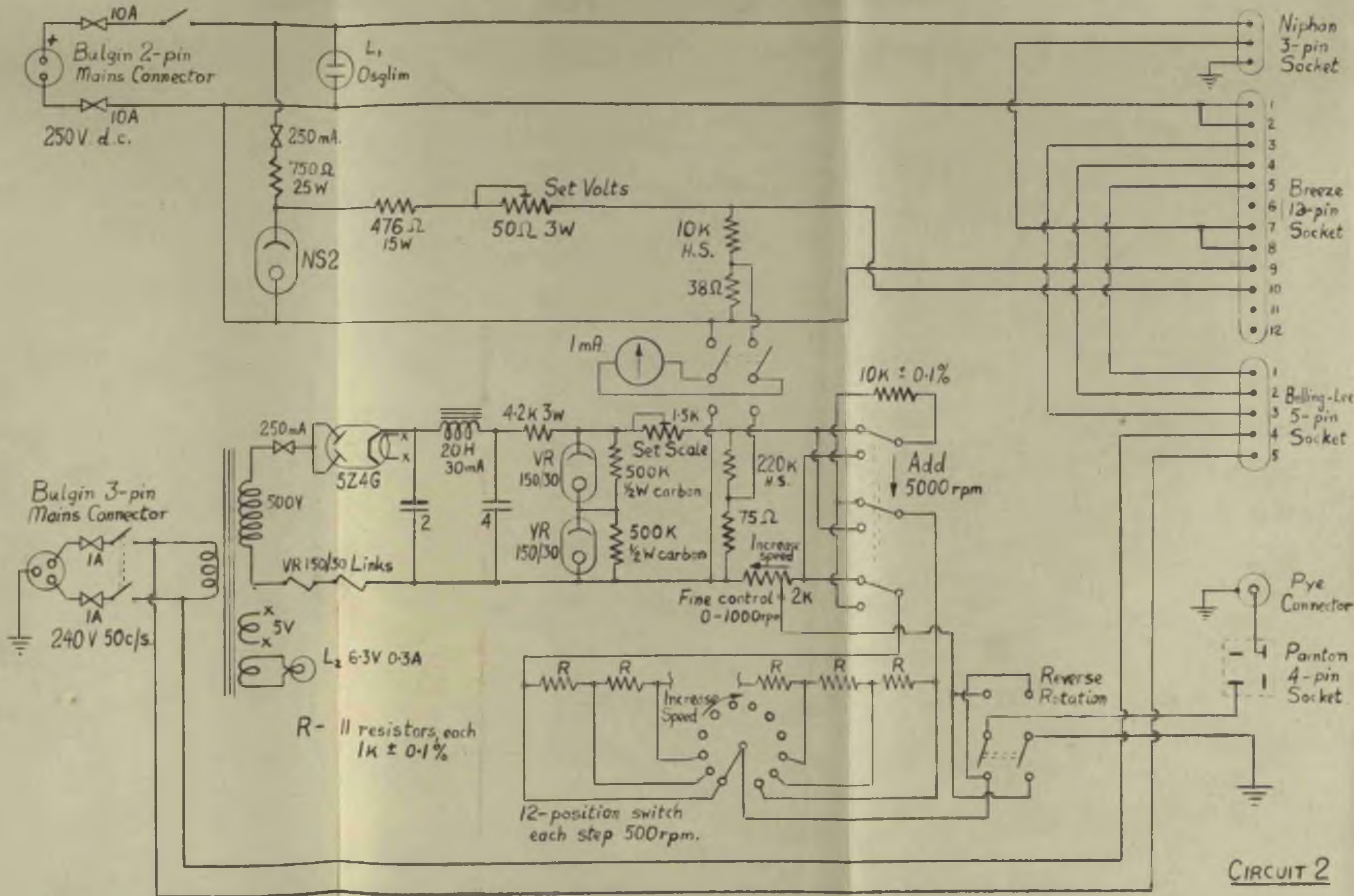
Conventions

1. "Ω" is normally omitted in resistance values. Thus 120, 12k and 1M represent 120Ω, 12kΩ and 1MΩ respectively.
2. A number beside the symbol for a capacitor represents its capacitance in μF. For any other unit of capacitance the appropriate abbreviation is used.

— CIRCUIT I —

CONTROL UNIT FOR EXCITER MARK III

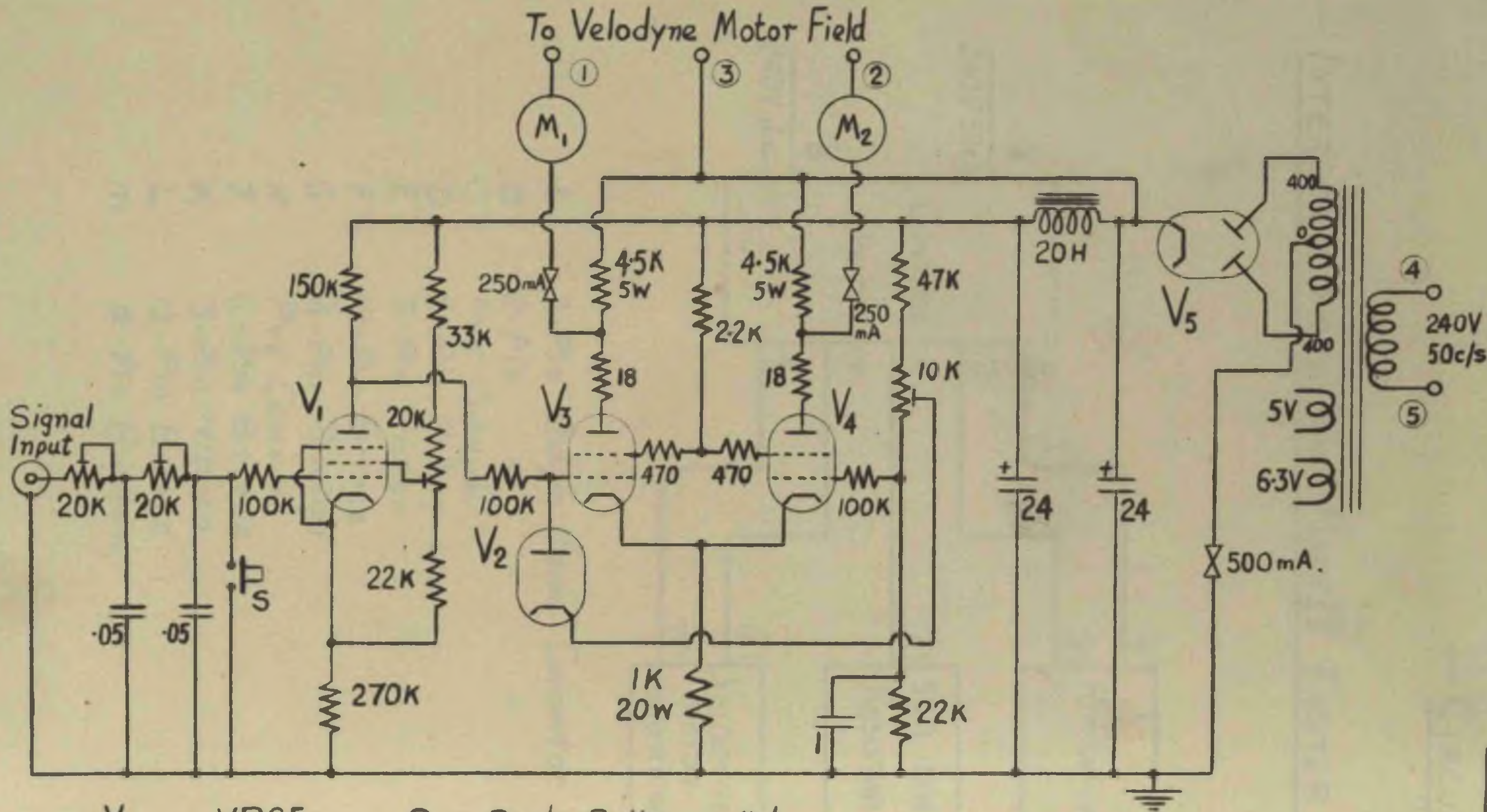




Note: All Resistors & Potentiometers wire-wound except when stated otherwise.

— CONTROL UNIT for MARK IV EXCITER SYSTEM —

CIRCUIT 2



V_1	VR65
V_2	VR92
V_3, V_4	807
V_5	5Z4G

S - *Push-Button switch for amplifier balancing*

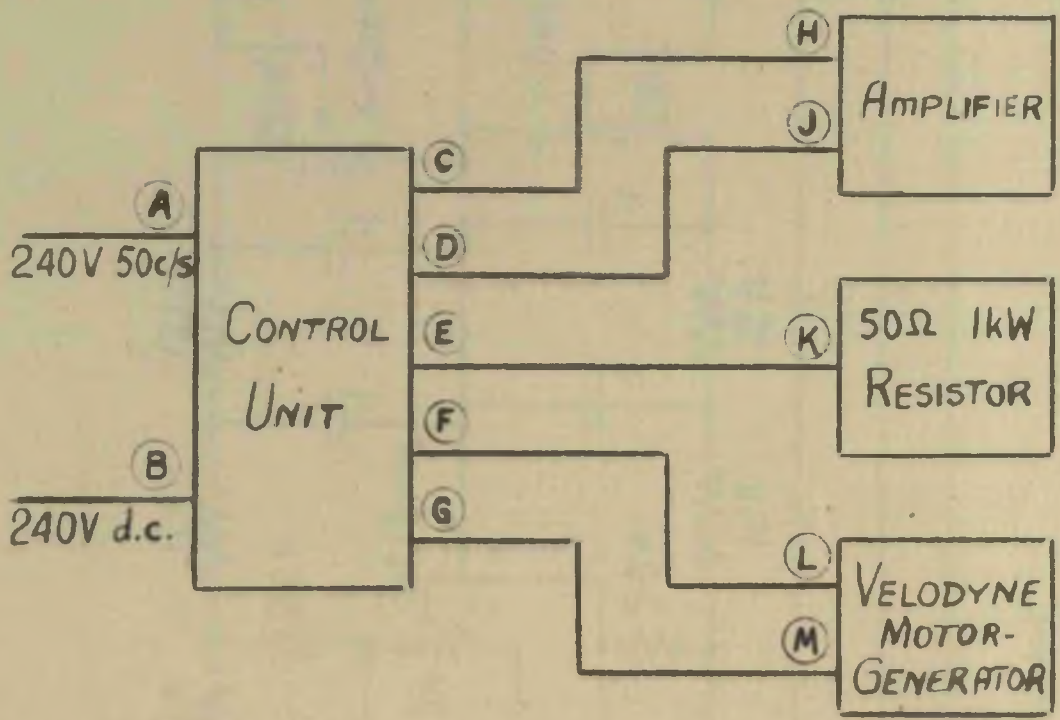
M_1, M_2 - 100mA F.s.d.

— D.C. AMPLIFIER —

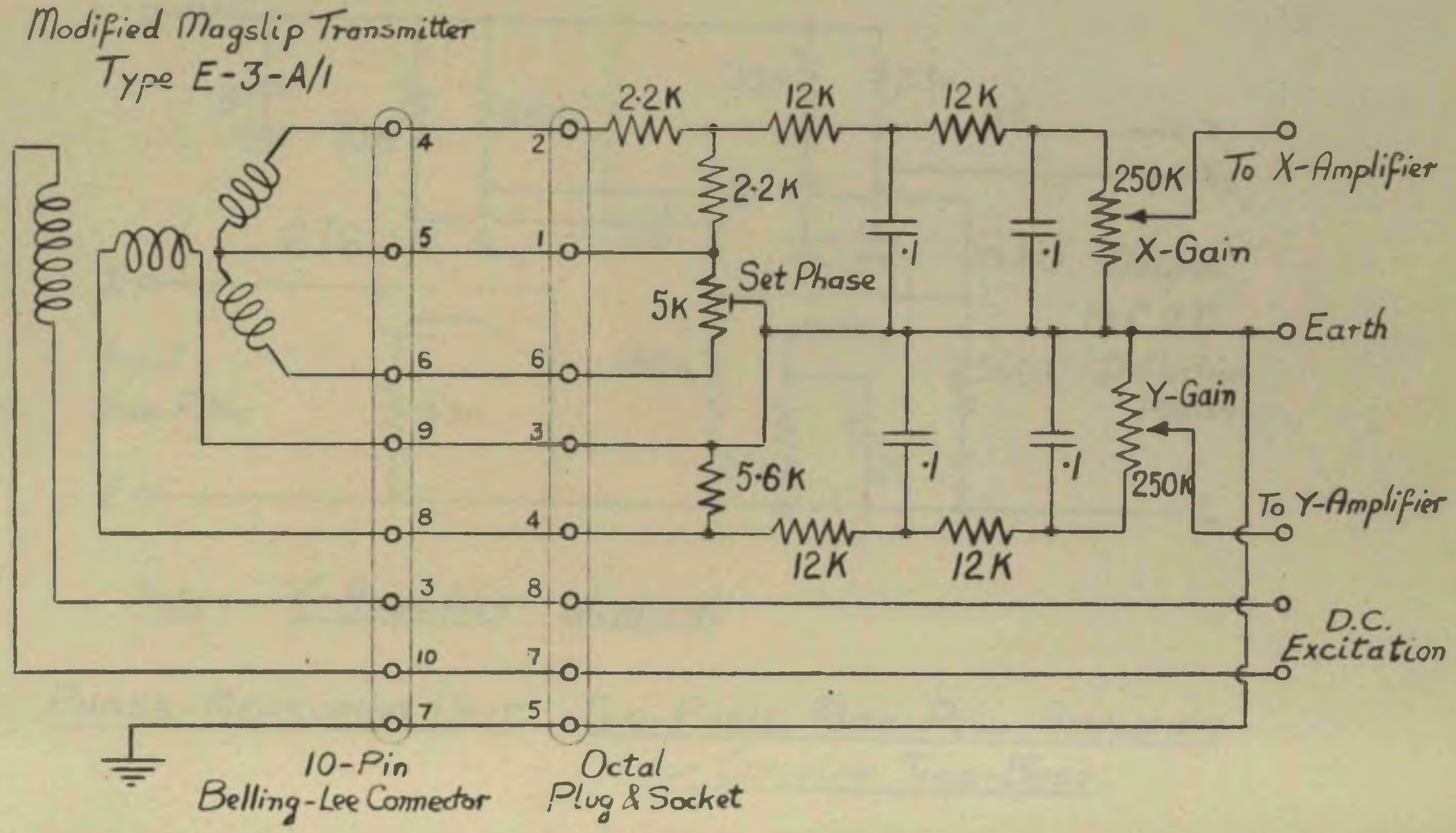
FOR

— MARK IV EXCITER SYSTEM —

— INTERCONNECTIONS FOR MARK IV EXCITER SYSTEM —

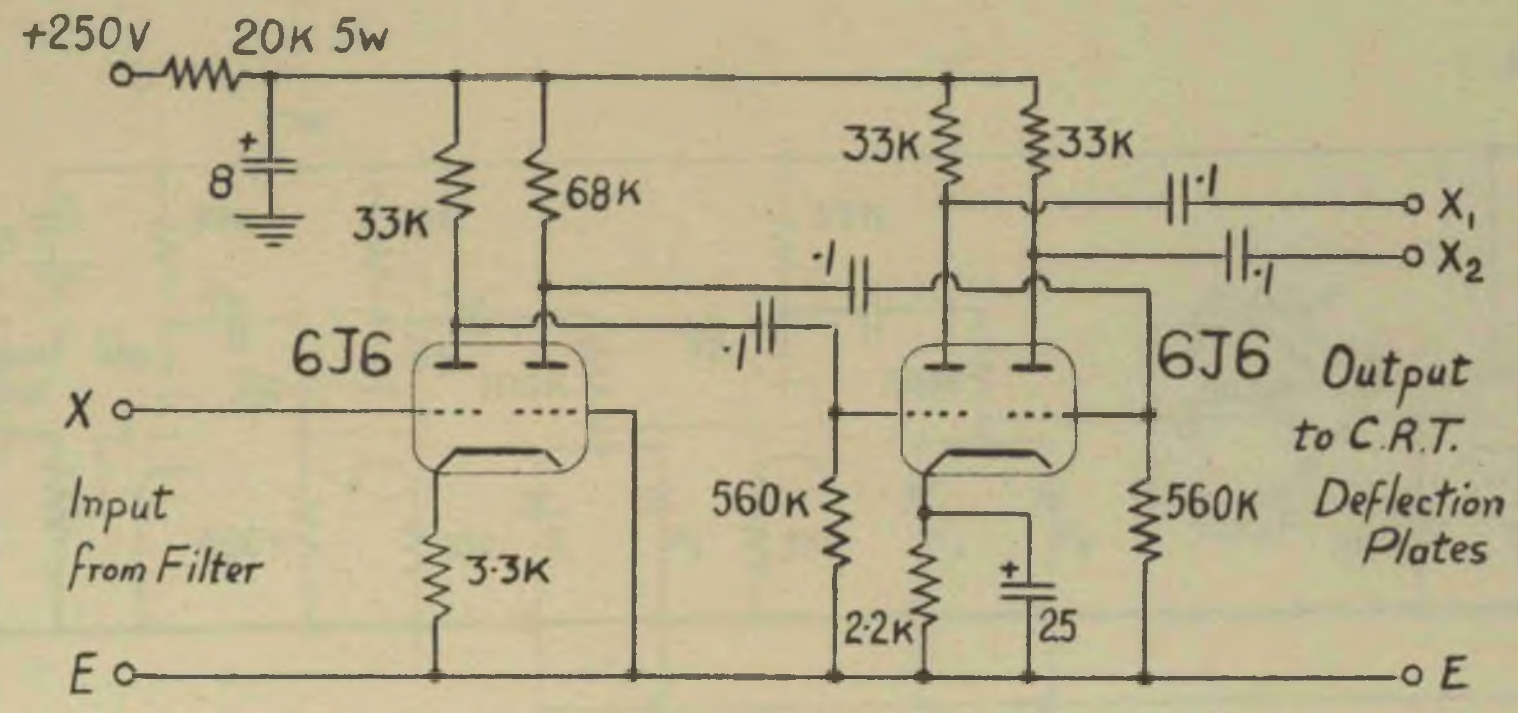


A	3-Pin Bulgin Mains Connector
B	2-Pin " " "
C	PYE COAXIAL
D	5-Pin Belling-Lee
E	3-Pin Niphan
F	12-Pin Breeze
G	4-Pin Painton
H	PYE COAXIAL
J	6-Pin Breeze
K	3-Pin Niphan
L	12-Pin Breeze
M	4-Pin Painton



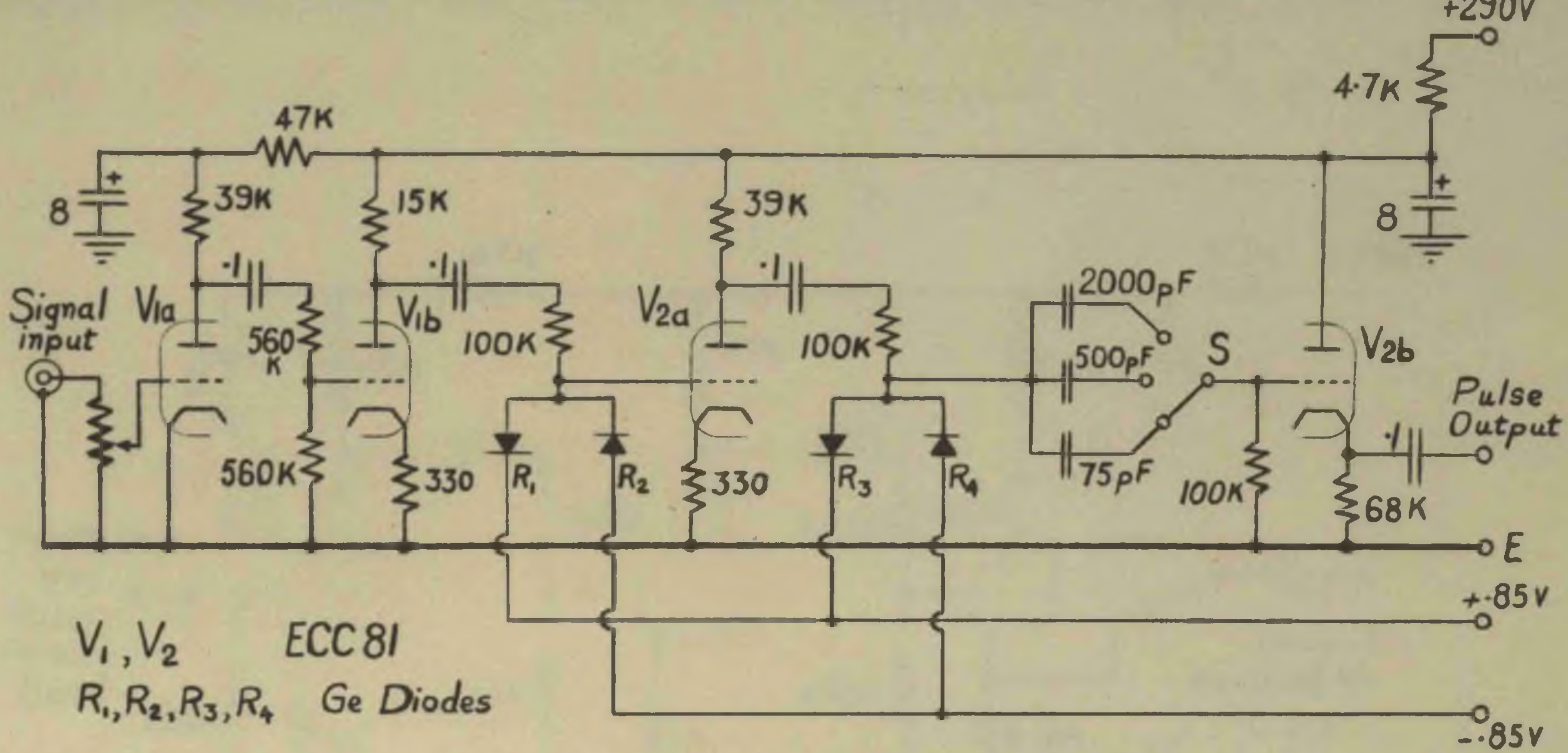
PHASE-MEASURING UNIT. TWO-PHASE GENERATOR & FILTER

— Circuit 6 —



Note: Y-Amplifier identical

Phase-Measuring Unit. Two-Phase Push-Pull Amplifier for Circular Time-Base.



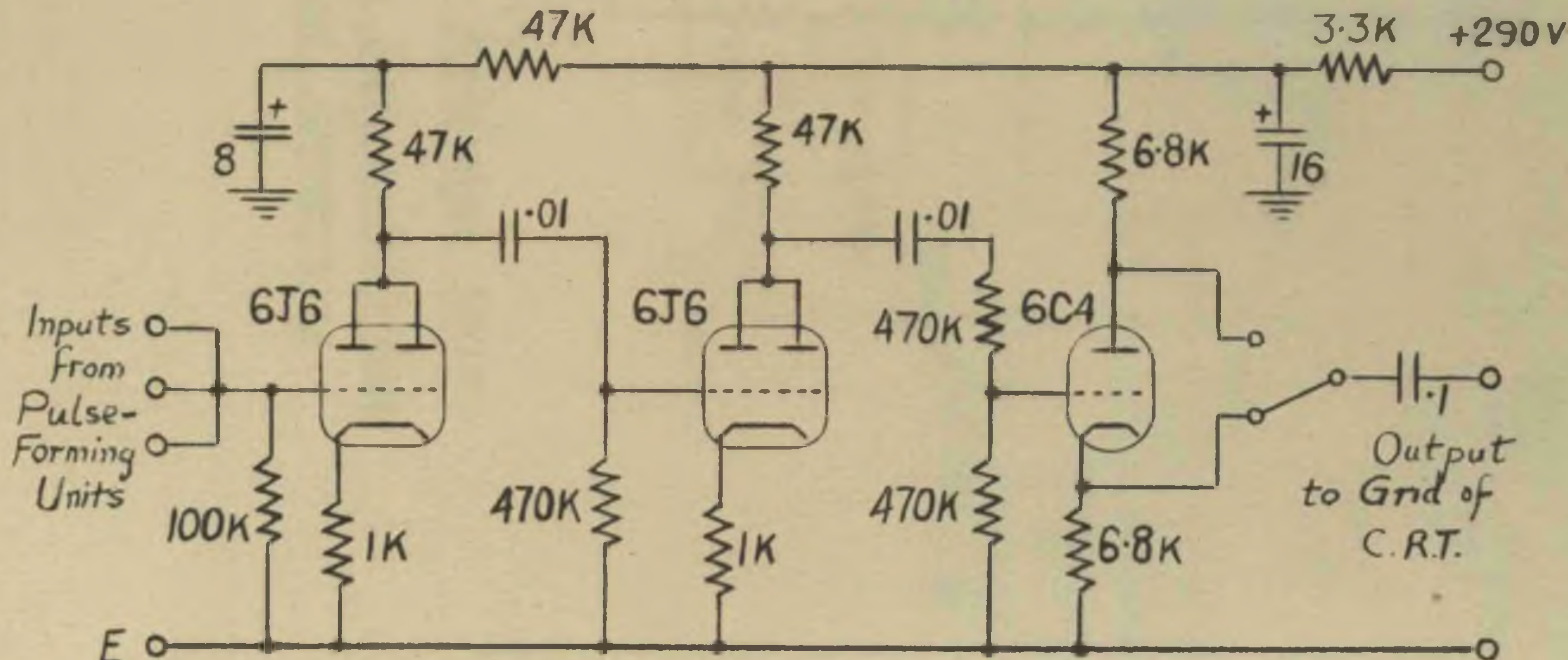
V_1, V_2 ECC 81

R_1, R_2, R_3, R_4 Ge Diodes

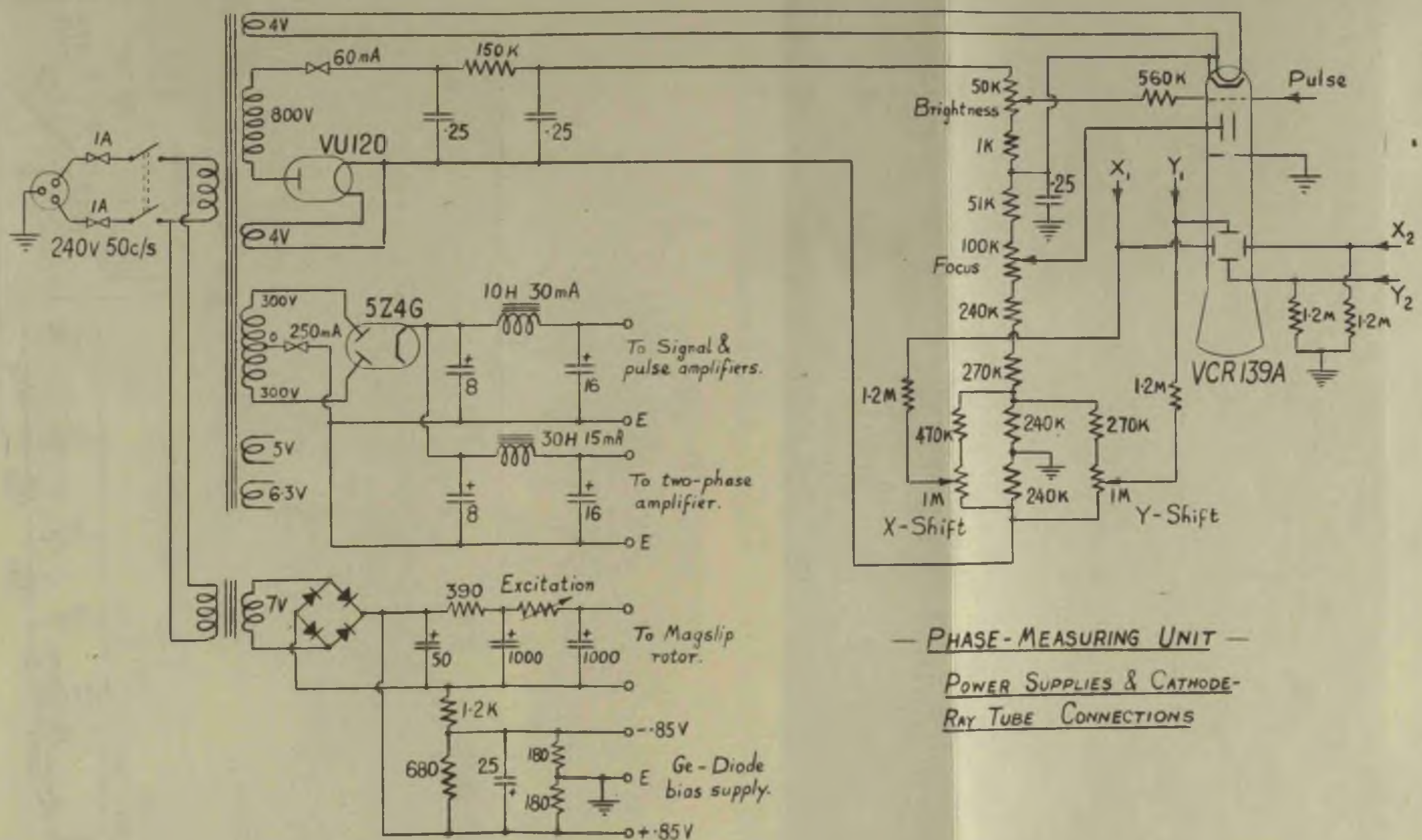
NOTE: Unit comprises three channels identical to the above.

— PHASE-MEASURING UNIT. SIGNAL AMPLIFIER & PULSE-FORMING UNIT —

— CIRCUIT 8 —

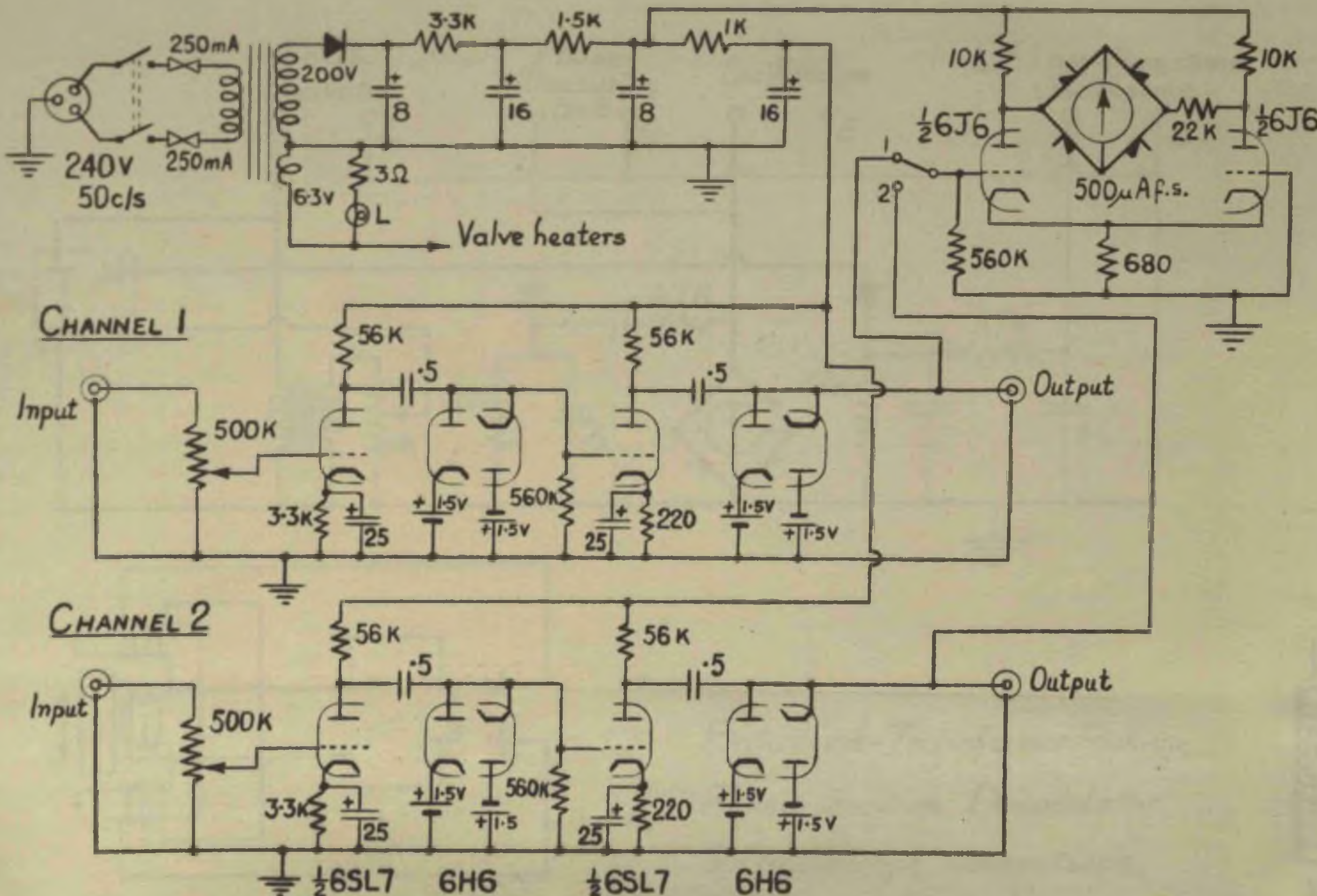


PHASE-MEASURING UNIT. PULSE AMPLIFIER

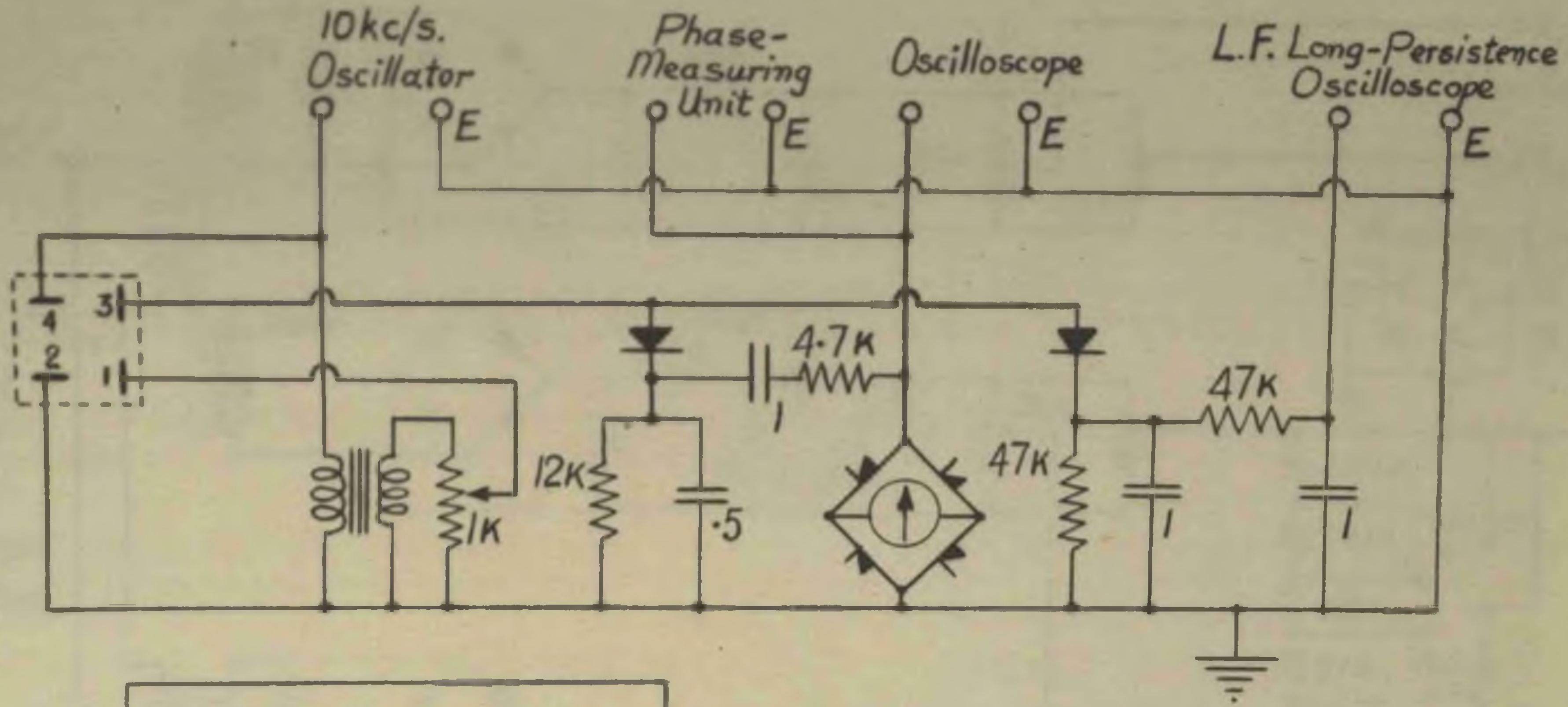


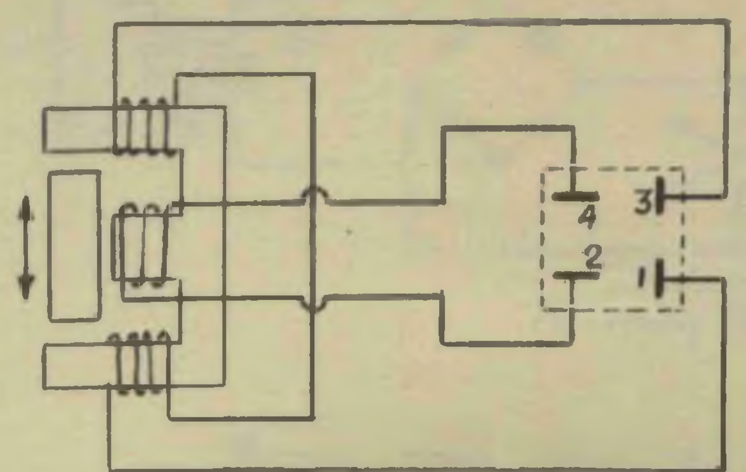
— PHASE-MEASURING UNIT —

POWER SUPPLIES & CATHODE-RAY TUBE CONNECTIONS



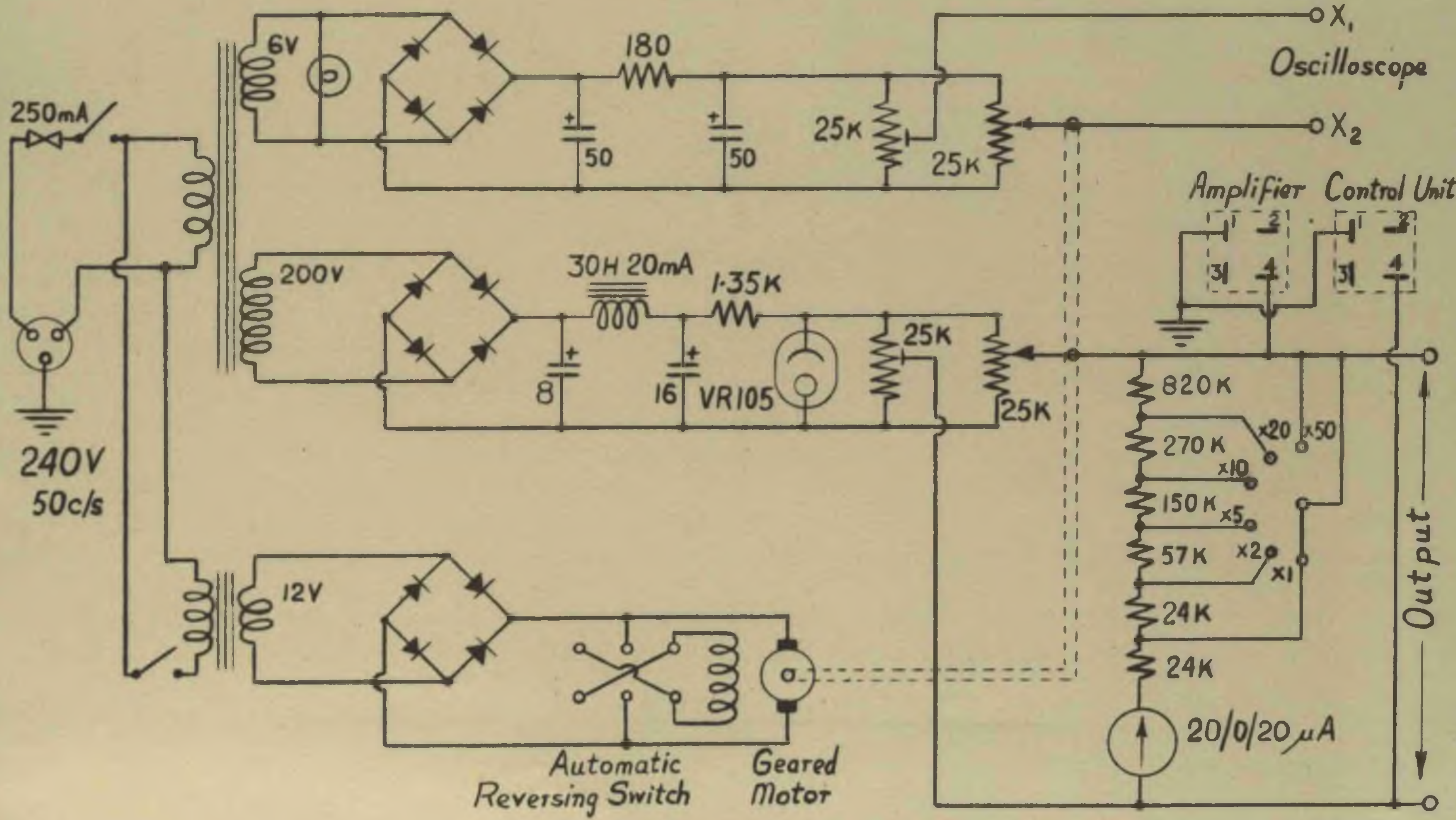
— TWO-CHANNEL LIMITER —





Balanced-Transformer Pick-up,
Phase-sensitive Demodulator
& Oscilloscope connections.

— Circuit 11 —



— MODULATOR UNIT & TIMEBASE FOR OSCILLOSCOPE —